Section 7.1 – Operations on Functions

Goals:

1. Find the sum, difference, product, and quotient of functions
2. To find the composition of functions.

I. Arithmetic Operations

A. Codes:

1. sum: $(f+g)(x)=$
2. difference: $(f-g)(x)=$
3. Product: $(f \cdot g)(x)=$
4. quotient: $\left(\frac{f}{g}\right)(x)=$

B. Examples

Given $f(x)=3x^2+7x$ and $g(x)=2x^2-x-1$ find:

1. $(f+g)(x)$

2. $(f-g)(x)$

Given $f(x)=2x^2+5x+2$ and $g(x)=3x^2+3x-4$ find:

3. $(f+g)(x)$

4. $(f-g)(x)$

Given $f(x)=3x^2-2x+1$ and $g(x)=x-4$ find:

5. $(f \cdot g)(x)$

6. $\left(\frac{f}{g}\right)(x)$

Given $f(x)=2x^2+3x-1$ and $g(x)=x+2$ find:

7. $(f \cdot g)(x)$

8. $\left(\frac{f}{g}\right)(x)$

II. Composition of Functions

A. Definition – When the _________ of one function is ___________ into another function to ___________ a ___________.

B. Examples of use

1. Converting $59°F$ to Kelvin

a) Convert $59°F$ to Celsius

b) Convert ______ to Kelvin

2. An $45 item on sale for 30% off and from 8am to 10am take an addition 50% off.

a) Take 30% off $45

b) Take 50% off __________

C. Notation: $f \circ g$ or $f(g(x))$ start with the *x* value in *g* and take its result and plug it into *f*.

D. Examples:

1. If $f(x)=\{(2,6),(9,4),(7,7),(0,-1)\}$ and $g(x)=\{(7,0),(-1,7),(4,9),(8,2)\}$ find $f \circ g$ and $g \circ f$.

2. If $f(x)=\{(1,2),(0,-3),(6,5),(2,1)\}$ and $g(x)=\{(2,0),(-3,6),(1,0),(6,7)\}$ find $f \circ g$ and $g \circ f$.

3. If $f(x)=3x^2-x+4$ and $g(x)=2x-1$ find $[f \circ g](x)$ and $[g \circ f](x)$

4. If $f(x)=x^2+2x+3$ and $g(x)=x+5$ find $[f \circ g](1)$ and $g(f(1))$

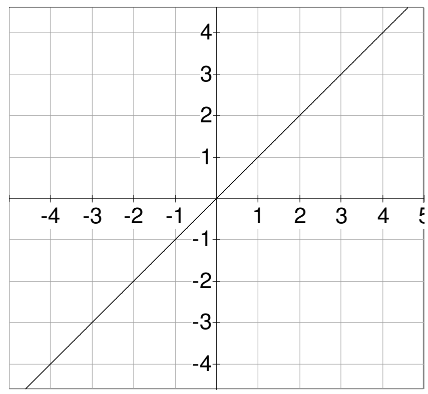
Homework: Day 1: p. 413 – 1, 2, 8, 11, 12, 16
Day 2: p. 413 – 3-7, 24, 27, 30, 37, 41-45 odds, 60, 70, 78

Section 7.2 – Inverse Functions and Relations

Goals:
1. To determine the inverse of a function or relation.
2. To graph a function and its inverse.

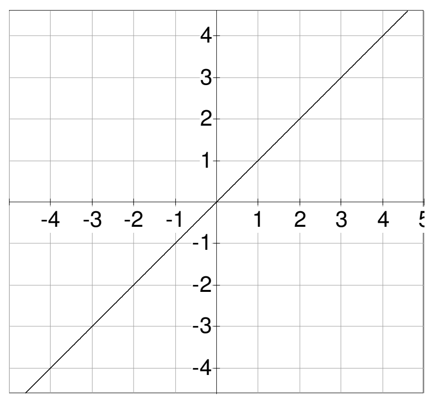
A. Definition of Inverse Relations

1. Algebraically: Two relations are _________ iff $[f \circ g](x) = x$ and $[g \circ f](x) = x$.
2. Geometrically: Two relations are inverses iff their __________ are ____________ over the line __________.
3. If *f* and f^{-1} are inverses, then $f(a) = b$ and $f^{-1}(b) = a$.
   [i.e., $f = (x, y)$ and $f^{-1} = (y, x)$]



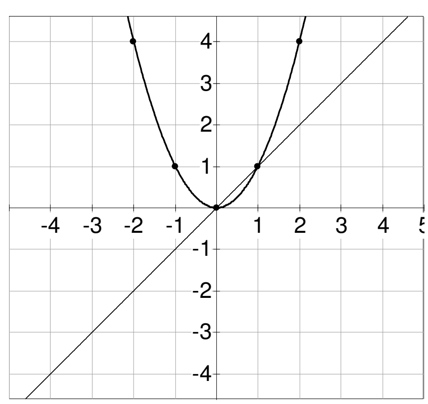
B. Examples:

1. Geometry The ordered pairs of the relation {(1, 3), (6, 3), (6, 0), (1, 0)} are the coordinates of the vertices of a rectangle. Find the inverse of this relation and determine whether the resulting ordered pairs are also the coordinates of the vertices of a rectangle.



2. a. Find the inverse of $f(x) = -\frac{1}{2}x + 1$
   b. Graph the inverse and the function.

3. Algebraically determine if the following are inverse functions:
   $f(x) = \frac{3}{4}x - 6$ and $g(x) = \frac{4}{3}x - 8$



4. Graph the inverse of the given graph.
   Is it a function?

Homework: p. 420 – 1-7 all, (15-30)/3, 39, 59-62 all

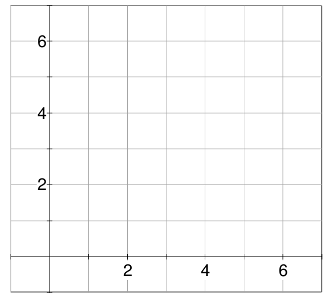
Section 7.3 – Square Root Functions and Relations

Goals:
1. To graph and analyze square root functions.
2. To graph square root inequalities.

I. Analyzing Square Root Functions and Graphing

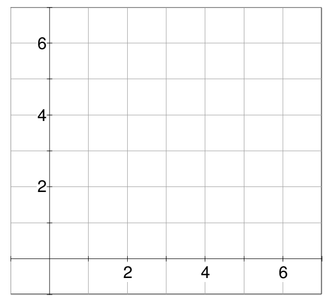
A. Analyzing/Graphing

1. Graphing ___________ numbers.
2. Determine the ______________ (_______________________________)
3. Determine the ______________ (_______________________________)
4. Determine the *x*- and *y*-intercepts
5. Example



a) State the domain and range of $f(x)=\sqrt{\frac{3}{2}x-1}$.

b) State the *x*- and *y*-intercepts of $f(x)=\sqrt{\frac{3}{2}x-1}$.

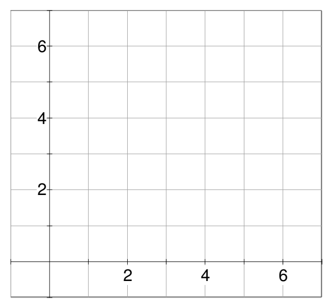


c) Graph: $f(x)=\sqrt{\frac{3}{2}x-1}$

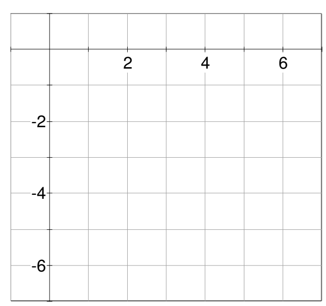
d) Graph $y=\sqrt{2x-2}$.
State the domain, range, and *x*- and *y*-intercepts.

II. Graphing Square Root Inequalities

A. Examples



1. Graph: $f(x)>\sqrt{3x+5}$



2. Graph: $f(x)\geq\sqrt{x}-6$

Homework: p. 427 – 1-10 all, 16, 17, 29, 30, 32, 60, 62

Goals:
1. To simplify roots having various indices.
2. To use a calculator to estimate roots of numbers.

I. General Information

A. Definition of Square Roots – For any real numbers *a* and *b*, if $a^2 = b$, then *a* is a square root of *b*.

B. Definition of n^th Roots – For any real numbers *a* and *b*, if $a^n = b$, then *a* is a n^th root of *b*.

C. Symbolism – $\sqrt[n]{x}$
  1. *n* is the Index
  2. *x* is the radicand

D. Even indices can have more than one n^th root.
  1. Examples
    a) 49 has two square roots.
    b) 16 has two 4^th roots.
  2. The nonnegative root is called the principal root.
    a) $\sqrt{36}$ is asking for the principle root.
    b) $-\sqrt{36}$ is asking for the opposite of the principle root.
    c) $\pm\sqrt{36}$ is asking for the both roots.

E. Odd indices will only have one n^th root
  1. $\sqrt[3]{8} = 2$
  2. $\sqrt[3]{-8} = -2$

F. Note: Roots cannot be done over addition (Ex. $\sqrt{a^2 + b^2}$)

II. Problems

A. Find each root
  1. $\pm\sqrt{16x^6}$
  2. $-\sqrt{(q^3+5)^4}$
  3. $\sqrt[5]{243a^{10}b^{15}}$
  4. $\sqrt{-4}$
  5. $\pm\sqrt{9x^8}$
  6. $-\sqrt{(a^3+2)^6}$
  7. $\sqrt[5]{32x^5y^{10}}$
  8. $\sqrt{-16}$
  9. $\sqrt[3]{27(x+2)^9}$

B. Calculator Approximations
  1. Designers must create satellites that can resist damage from being struck by small particles of dust and rocks. A study showed that the diameter in millimeters *d* of the hole created in a solar cell by a dust particle traveling with energy *k* in joules is about $d = 0.926\sqrt[3]{k} - 0.169$. Estimate the diameter of a hole created by a particle traveling with energy 3.5 joules.

  2. $\sqrt[5]{12589}$
  3. $\sqrt[7]{1537}$

Homework: p. 433 – 1-11 all, (12-36)/3, 75, 79, 81, 85, 86

Section 7.5A – Operations with Radical Expressions

Goals:
1. To simplify radical expressions using multiplication and division.
2. To rationalize the denominator of a fraction containing a radical

I. Radical Expressions

A. Properties

1. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$
2. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

B. Examples on Simplifying

1. $-\sqrt{30a^3}$
2. $\pm\sqrt{64x^2y^3z}$
3. $-\sqrt{54x^4y^5z^7}$
4. $\sqrt[3]{54a^3b^7}$
5. $\sqrt[4]{32a^8b^6}$
6. $\sqrt{60xy^3}$
7. $2\sqrt{2} \cdot 4\sqrt{6}$
8. $\sqrt{10x^2y} \cdot \sqrt{40xy^3}$
10. $\sqrt{12m^2n} \cdot \sqrt{6mn}$
10. $\sqrt[4]{4x^3} \cdot \sqrt[4]{8x^2y^5}$
11. $\sqrt{\dfrac{5}{4}}$
12. $\sqrt[3]{\dfrac{16}{125}}$
13. $\dfrac{20\sqrt{8}}{2\sqrt{2}}$

II. Rationalizing the Denominator

A. Protocol – In math the proper protocol is not to have any root in the denominator.

B. Examples

1. $\dfrac{1}{\sqrt{2}}$
2. $\dfrac{5}{7\sqrt{3}}$
3. $\sqrt[3]{\dfrac{2}{3}}$
4. $\dfrac{3}{\sqrt[4]{2}}$

Homework: worksheet

Algebra 2

Worksheet Section 7.5A

Simplify the following using exact values only.

1. $\sqrt[3]{-432}$
2. $\sqrt{540}$
3. $\sqrt{5}\left(\sqrt{10}-\sqrt{45}\right)$
4. $\sqrt[3]{6}\left(4\sqrt[3]{12}+5\sqrt[3]{9}\right)$
5. $\left(2\sqrt[3]{24}\right)\left(7\sqrt[3]{18}\right)$
6. $\sqrt[4]{32x^4y^5n^{10}}$
7. $\sqrt{1792}$
8. $\sqrt[3]{-6750}$
9. $\sqrt{3x^2y^3}\cdot\sqrt{75xy^5}$
10. $\sqrt[3]{9t^5v^8}\cdot\sqrt[3]{6tv^4}$
11. $\sqrt{60}\cdot\sqrt{105}$
12. $\sqrt[3]{3600}\cdot\sqrt[3]{165}$
13. $\dfrac{\sqrt{35}}{\sqrt{7}}$
14. $\dfrac{\sqrt[4]{42}}{\sqrt[4]{7}}$
15. $\sqrt{\dfrac{3}{5}}$
16. $\sqrt{\dfrac{6}{x}}$
17. $\sqrt[4]{\dfrac{5}{27}}$
18. $\dfrac{\sqrt[4]{8}}{\sqrt[4]{9a^3}}$
19. $\sqrt{\dfrac{20}{5}}$
20. $\dfrac{\sqrt{11}}{\sqrt{9}}$
21. $\sqrt[3]{\dfrac{2}{9}}$
22. $\sqrt[3]{\dfrac{9}{25}}$
23. $\dfrac{\sqrt[3]{16}}{\sqrt[3]{4}}$
24. $\sqrt[3]{\dfrac{9}{4}}$
25. $\dfrac{\sqrt{22}}{\sqrt{2}}$
26. $\dfrac{7}{\sqrt[3]{9}}$
27. $\sqrt[4]{112x^5}$
28. $\sqrt[4]{a^5b^3}\cdot\sqrt[4]{81a^3b^2}$
29. $\sqrt{3x^2y^3}\cdot\sqrt{15x^2y}$

Section 7.5B – Operations with Radical Expressions

Goals:
1. To add, subtract, multiply, and divide radical expressions

I. Adding and Subtracting Radicals

A. Note:

1. To add radicals they must be like radical expressions (i.e., both the indices and radicands are identical).
1. Treat radicals like they are variables.

B. Examples

1. $2\sqrt{3}+5+7\sqrt{3}-2$
2. $10\sqrt{2}-3\sqrt{2}+7+6\sqrt{2}$
3. $3\sqrt{27}-7\sqrt{3}-12$
4. $5\sqrt{6}-3\sqrt{24}+\sqrt{150}$
5. $\sqrt[3]{16a}+4\sqrt[3]{54a}$
6. $5\sqrt[3]{40x}-7\sqrt[3]{5x}$

II. Multiplying Radicals

Examples

1. $\left(\sqrt{6}+\sqrt{3}\right)\left(\sqrt{3}+\sqrt{2}\right)$
2. $\left(2\sqrt{3}+4\sqrt{5}\right)\left(\sqrt{3}+6\sqrt{5}\right)$
3. $\left(12+\sqrt{3}\right)\left(12-\sqrt{3}\right)$
4. $\left(4\sqrt{5}+2\sqrt{7}\right)\left(4\sqrt{5}-2\sqrt{7}\right)$

III. Rationalizing the Denominator

Examples

1. $\dfrac{2+\sqrt{6}}{2-\sqrt{6}}$
2. $\dfrac{1+2\sqrt{5}}{6-\sqrt{5}}$

Homework: worksheet

Algebra 2

Worksheet Section 7.5B

Simplify the following using exact values only.

1. $4\sqrt{24}+\sqrt{18}-5\sqrt{24}-4\sqrt{450}$

2. $\sqrt{45}-\left(\sqrt{5}\right)^2+\sqrt{180}$

3. $\sqrt[3]{56}+\sqrt[3]{24}-\sqrt{28}$

4. $9\sqrt[4]{5}-\sqrt[4]{5}+11\sqrt[4]{5}$

5. $\sqrt[4]{x^8}+2\sqrt[3]{x^6}-\sqrt{x^2}+\sqrt[3]{x^3}$

6. $\sqrt{75v^5t^3}-\sqrt{48v^3t^7}$

7. $\left(6-\sqrt{3}\right)^2$

8. $\left(4\sqrt{7}+5\sqrt{2}\right)\left(2\sqrt{7}-3\sqrt{2}\right)$

9. $2\sqrt{48}-\sqrt{12}-3\sqrt{63}+\sqrt{112}$

10. $\sqrt[3]{216}-\sqrt[3]{48}+\sqrt[3]{432}$

11. ** $\dfrac{3}{2-\sqrt{5}}$

12. ** $\dfrac{6}{\sqrt{2}-1}$

13. ** $\dfrac{5+\sqrt{3}}{4+\sqrt{3}}$

14. ** $\dfrac{1-\sqrt{3}}{5+\sqrt{3}}$

15. ** $\dfrac{6}{2-\sqrt{7}}$

16. ** $\dfrac{7}{4-\sqrt{3}}$

17. $\dfrac{\sqrt{x+1}}{\sqrt{x-1}}$

18. Challenge: $\sqrt[3]{144}+\sqrt[3]{\dfrac{2}{3}}-5\sqrt[3]{18}$

19. Challenge: $\sqrt{\dfrac{3}{8}}+\sqrt{54}-\sqrt{6}$

20. Challenge: $\sqrt{\dfrac{2}{5}}+\sqrt{40}-\sqrt{10}$

Section 7.6 – Rational Exponents

Goals:
1. To write expressions with rational exponents in simplest radical form and vice versa.
2. To evaluate expressions in either exponential or radical form

I. Radicals in Exponential Form

A. Rules:

1. $\sqrt[n]{b} = b^{\frac{1}{n}}$
2. $\sqrt[n]{b^m} = b^{\frac{m}{n}}$

B. Examples

1. $36^{\frac{1}{2}}$

2. $64^{\frac{1}{3}}$

3. $81^{\frac{1}{4}}$

4. $49^{-\frac{1}{2}}$

5. $\left(\frac{1}{8}\right)^{-\frac{1}{3}}$

6. $36^{\frac{3}{2}}$

7. $64^{\frac{5}{6}}$

8. $27^{\frac{2}{3}} \cdot 27^{\frac{2}{3}}$

II. Calculator

Examples

1. $194481^{\frac{3}{4}}$

2. $256^{\frac{5}{6}}$

3. $117649^{0.325}$

Homework: p. 449 – 1-14 all, 16-28 all, 36, 37, 48, 49, 61, 75-77, all, 87, 92, 95

Section 7.7 – Solving Radical Equations and Inequalities

Goals:

1. To solve equations and inequalities containing radicals.

Examples

1. $\sqrt{y-2}-1=5$
2. $\sqrt{3t-2}+7=3$
3. $\sqrt{x-3}-2=6$
4. $\sqrt{x-12}=2-\sqrt{x}$
5. $\sqrt{x+5}=-1-\sqrt{x}$
6. $(3y+1)^{\frac{1}{3}}+5=0$
7. $\sqrt[3]{2y+1}-3=0$
8. $7\left(\sqrt[6]{5m+4}\right)-4=10$
9. $\sqrt{3x-6}+4\le 7$
10. $\sqrt{2x+5}-2\le 9$

Homework: p. 456 – 1-22 all, 67, 81-89 odds