Section 8.1 –Exponential Functions

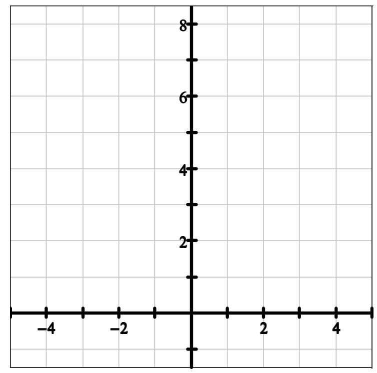
Goals:

1. To simplify expressions and solve exponential equations involving real exponents.

I. Definition of Exponential Function

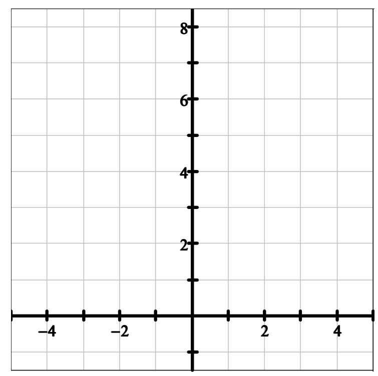
An _______________ function is in the form _____________,
where __________ and __________.

II. Important Information



A. General Equation: $y = ab^x$
B. Initial Value
C. Growth vs. Decay
   1. Growth
      a)
      b)
   2. Decay
      a)
      b)

III. Graphing Exponential Functions



A. Graph the growth function: $y = 4^x$
   1. What is the domain and range of the function?
   2. Any asymptotes?

B. Graph the decay function: $y = \left(\frac{1}{5}\right)^x$
   1. What is the domain and range of the function?
   2. Any asymptotes?

IV. Growth or Decay

A. Determine whether the following are exponential growth or decay.

1. $y = (0.7)^x$
2. $y = \frac{1}{2}(3)^x$
3. $y = 10\left(\frac{4}{3}\right)^x$
4. $y = (0.5)^x$
5. $y = \frac{1}{3}(2)^x$
6. $y = 10\left(\frac{2}{5}\right)^x$

B. Applications

1. In 2006, there were 1,020,000,000 people worldwide using the Internet. At that time, the number of users was growing by 19.5% annually. Write an equation representing the number of users from 2006 to 2016, if that rate continued.

2. The pressure of the atmosphere is 14.7 lb/in^2 at Earth's surface. It decreases by about 20% for each mile of altitude up to about 50 miles. Write an equation that estimates the atmospheric pressure at an altitude of t miles. Estimate the atmospheric pressure at an altitude of 10 miles.

3. The pressure of a car tire with a bent rim is 34.7 lb/in^2 at the start of a road trip. It decreases by about 3% for each mile driven due to a leaky seal. Write an equation that estimates the air pressure of the tire after t miles. Estimate the air pressure of the tire after 20 miles.

Homework: Worksheet

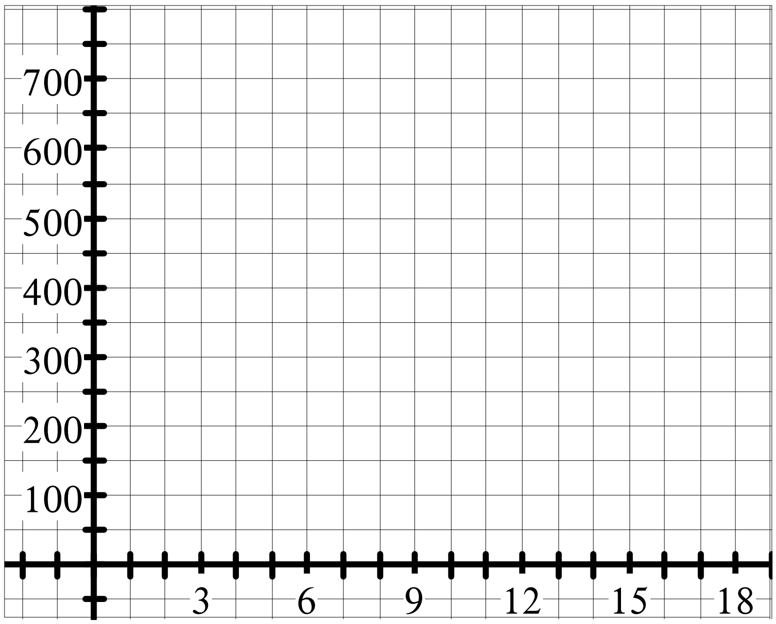
Section 8.1B – Modeling using Exponential Functions

I. Work Together

A. In previous lessons you have learned how to find a line of best fit for a set of data. Some data sets are better modeled by exponential functions.

1. Draw a scatter plot for the given data.

2. [0,18] scale: 1 x [0,800] scale: 50



US Sales of Compact Disks	
Year	**Millions of CD's**
1987 – 7	102.1
8	149.7
9	207.2
10	286.5
11	333.3
12	407.5
13	495.4
1994 – 14	662.1

3. Find the exponential regression equation (ExpReg) that best fits the data and graph.

4. Write a sentence that describes the equation and data.

5. Based on the graph find the number of CDs sold in 2000.

6. Do you think a linear regression line would be a good model for the data? Why or why not? (Hint: Explain the difference between the two regression curves.)

B. Wrap Up

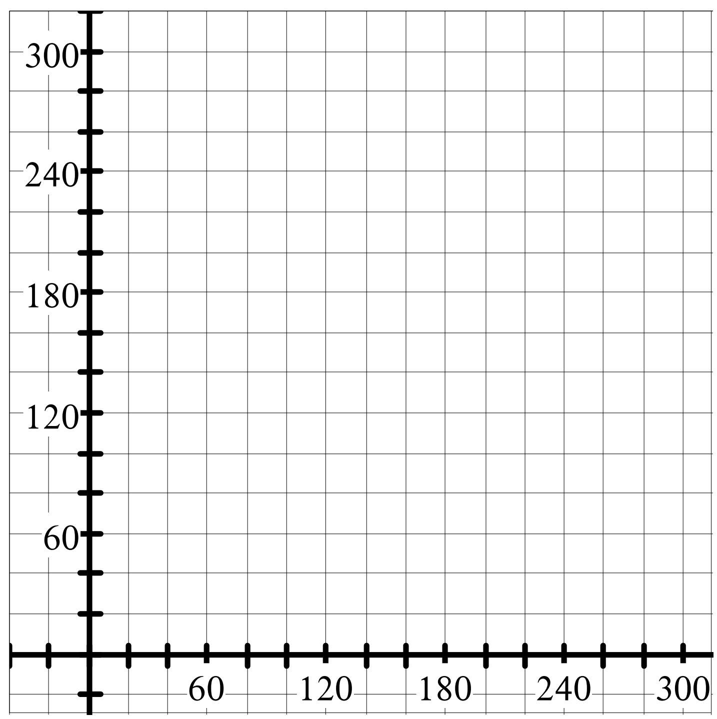
Homework: Worksheet

Worksheet
Section 8.1B: Modeling using Exponential Functions
Name: ___________________________

Years Since 1780	**Population (In millions)**	**Years Since 1780**	**Population (In millions)**	**Years Since 1780**	**Population (In millions)**
10	3.9	80	31.4	150	132.2
20	5.3	90	38.6	160	132.2
30	7.2	100	50.2	170	151.3
40	9.6	110	63.0	180	179.3
50	12.8	120	76.2	190	203.3
60	17.0	130	92.2	200	226.5
70	23.2	140	106.1	210	248.7

1. Use a graphing calculator to draw a scatter plot of the data. Then calculate and graph the curve of best fit that shows how the year is related to the population. Use ExpReg for this problem.

   Use: [0.300] scale:20 x [0,300] scale:20



2. Write the equation of best fit.

3. Write a sentence that describes the equation and data.

4. Based on the graph, estimate the population for 2000. Explain how you found your answer.

5. Based on the graph, when will the population reach 325 million? Explain how you found your answer.

Section 8.2 – Solving Exponential Equations and Inequalities

Objectives:

1. To solve exponential equations and inequalities.

I. Solving Exponential Equations in the form $b^x = b^y$

A. Property: Let $b > 0$ and $b \neq 1$. Then ____________ iff ___________.

B. Examples:

1. $3^x = 9^4$
2. $2^{5x} = 4^{2x-1}$
3. $3^{2x} = 9^{5x-4}$

C. Writing an exponential function:

Example:

1. In 2000, the population of Phoenix was 1,321,045. By 2007, it was estimated at 1,512,986. Write an exponential function that could be used to model the population of Phoenix.
   a) Write *t* in terms of the numbers of years since 2000.

   b) Predict the population of Phoenix in 2013.

II. Solving Exponential Inequalities in the form $b^x < b^y$ or $b^x > b^y$

A. Property: Let $b > 1$. Then __________ iff _________ and __________ iff __________.

B. Examples:

1. $5^{3-2x} > \frac{1}{625}$
2. $3^{4-3x} > \frac{1}{243}$

Homework: p. 488 – 1-8 all, 10, 11, 16, 17, 20, 21, 23, 25, 43, (54-69)/3

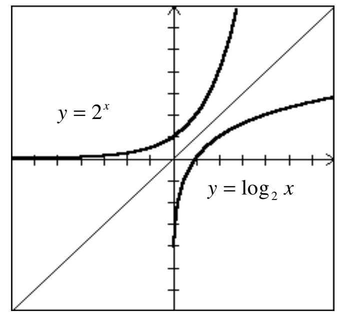
Section 8.3 – Logarithms and Logarithmic Functions

Objectives:
1. Evaluate logarithmic functions.
2. Graph logarithmic functions

I. Logarithmic Functions and Expressions

A. Write the following in inverse form:

1. $y = 3x$ 2. $y = x^2$ 3. $y = x + 3$ 4. $y = 2^x$



B. Important Information

1. Inverse function of exponential functions $x = b^y$
2. Notation: If __________, then __________.
3. Equivalence Statement:
4. Question asked for by the symbol $\log_b x$:
   "*b* to what ________ will equal *x*?"

C. Examples:

1. Logarithmic to Exponential Form

   a) $\log_3 9 = 2$ b) $\log_{10} \frac{1}{100} = -2$ c) $\log_2 8 = 3$

2. Exponential to Logarithmic Form

   a) $5^3 = 125$ b) $27^{\frac{1}{3}} = 3$ c) $3^4 = 81$

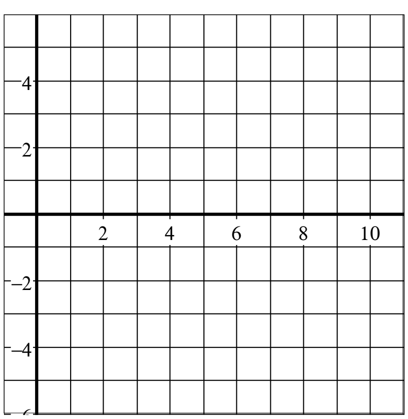
3. Evaluate Logarithmic Expressions

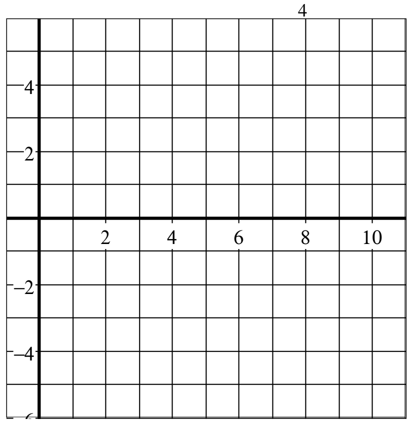
   a) $\log_3 243$ b) $\log_{10} 1000$

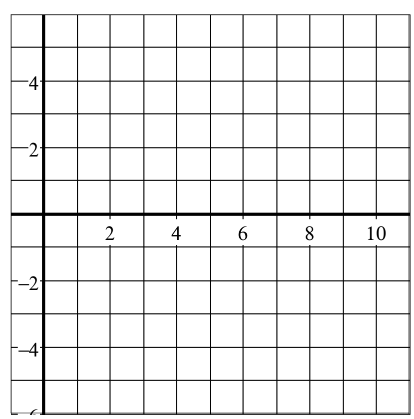
II. Graphing Logarithmic Functions

A. Examples:

1. $f(x) = \log_3 x$ 2. $f(x) = \log_{\frac{1}{4}} x$ 3. $f(x) = \log_5 x$







Homework: p. 496 – 1-9 all, 12, (15-36)/3, 37, 38, 41, 50, 65, 72, 73

Section 8.4 – Solving Logarithmic Equations and Inequalities

Objectives:
1. To solve equations involving logarithms.
2. To solve inequalities involving logarithms.

I. Logarithmic Equations
Examples: Solve the following Logarithmic Equations

1. $\log_8 x = \frac{4}{3}$

2. $\log_{27} n = \frac{2}{3}$

Key Concept — For Your FOLDABLE

Property of Equality for Logarithmic Functions

Symbols If b is a positive number other than 1, then $\log_b x = \log_b y$ if and only if $x = y$.

Example If $\log_5 x = \log_5 8$, then $x = 8$. If $x = 8$, then $\log_5 x = \log_5 8$.

3. $\log_4 x^2 = \log_4 (-6x - 8)$

4. $\log_4 x^2 = \log_4 (x + 20)$

II. Logarithmic Inequalities

A. Property 1

Key Concept — For Your FOLDABLE

Property of Inequality for Logarithmic Functions

If $b > 1$, $x > 0$, and $\log_b x > y$, then $x > b^y$.

If $b > 1$, $x > 0$, and $\log_b x < y$, then $0 < x < b^y$.

B. Examples

1. $\log_6 x > 3$

2. $\log_3 x < 2$

C. Property 2

Key Concept — For Your FOLDABLE

Property of Inequality for Logarithmic Functions

Symbols If $b > 1$, then $\log_b x > \log_b y$ if and only if $x > y$, and $\log_b x < \log_b y$ if and only if $x < y$.

Example If $\log_6 x > \log_6 35$, then $x > 35$.

D. Examples:

1. $\log_7 (2x + 8) > \log_7 (x + 5)$

2. $\log_7 (4x + 5) < \log_7 (5x + 1)$

Homework: Day 1: p. 504 – 1-8 all, 11, 12, 15, 16, 25, 33, 34, 38, (51-66)/3
Day 2: p 508 – 1, 2-8 all, 12-24 all

Section 8.5 – Properties of Logarithms

Objectives:

1. To simplify and evaluate expressions using properties of logarithms
2. To solve equations involving logarithms

I. Logarithmic Properties

A. Properties:

1. $\log_b mn =$
2. $\log_b \frac{m}{n} =$
3. $\log_b m^p = p \cdot \log_b m$
4. $\log_b b^m =$ or $b^{\log_b m} =$

B. Examples: Given $\log_2 3 = 1.5850$ and $\log_2 5 = 2.3219$

1. Find: $\log_2 8$
2. Find: $\log_2 \frac{4}{3}$
3. Find: $\log_2 36$
4. Find: $\log_2 \frac{25}{9}$
5. Find: $\log_2 32$
6. Find: $\log_2 \frac{64}{15}$

C. Solve

1. $\log_3(4x+5) - \log_3(3-2x) = 2$
2. $\log_5(x^2+3) - \log_5(x-1) = \log_5 7$
3. $4\log_8 x = \log_8 81$

Homework: p. 512 – 1-4 all, 6-11 all, 12-17 all, 19-21 all, 23-26 all, 51-58 all

Section 8.6 – Common Logarithms

Objectives:
1. To find common logarithms and antilogarithms.
2. To solve problems involving common logarithms.
3. To solve exponential equations and inequalities.

I. The common logarithm(log) is $\log_{10}$

Examples
1. log 1000 =
2. log 100 =
3. log 0.01 =
4. log 27 =

II. Antilogarithms

A. Antilogarithm means to apply the inverse of a logarithm.

B. What is the inverse of $y = \log_{10} x$?

C. Examples
1. $3 = \log x$
2. $0.2568 = \log x$
3. The loudness *L*, in decibels, of a sound is $L = 10\log\left(\frac{I}{m}\right)$ where *I* is the intensity of the sound and *m* is the minimum intensity of sound detectable by the human ear. The sound of a jet engine can reach a loudness of 125 decibels. How many times the minimum intensity of audible sound is this, if *m* is defined to be 1?

III. Solve Exponential Equations and Inequalities Using Logarithms

Examples
1. $5^x = 62$
2. $3^x = 17$
3. $3^{7x} > 2^{5x-3}$
4. $5^{3x} < 10^{x-2}$

IV. Converting logarithms: $\log_b a = \frac{\log_m a}{\log_m b}$

Examples:
1. $\log_5 140$
2. $\log_5 16$

Homework: p. 519 – 1-15 all, 22, (24-39)/3, 68, 77-83 odds

Section 8.7 – Base *e* and Natural Logarithms

Objectives;

1. To evaluate expressions involving the natural base and natural logarithm.
2. To solve exponential equations and inequalities using natural logarithms.
4. To solve problems involving natural logarithms and *e*.

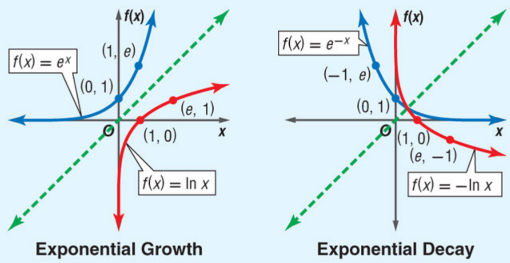
I. Natural Base Functions

A. Key Concept

Key Concept **Natural Base Functions** For Your FOLDABLE

The function $f(x) = e^x$ is used to model continuous exponential growth.
The function $f(x) = e^{-x}$ is used to model continuous exponential decay.

The inverse of a natural base exponential function is called the **natural logarithm**. This logarithm can be written as $\log_e x$, but is more often abbreviated as $\ln x$.



B. Equivalent Statement:

C. Examples:

1. $e^x = 23$ 2. $e^4 = x$ 3. $\ln x \approx 1.2528$ 4. $\ln 25 = x$

II. Natural Logarithmic Properties

A. Same as any other logarithm

B. Examples: Write as a single natural logarithm

1. $4\ln 3 + \ln 6$ 2. $2\ln 3 + \ln 4 + \ln y$

III. Solve Exponential Equations and Inequalities Using Natural Logarithms

A. Examples

1. $3e^{-2x} + 4 = 10$ 2. $2\ln 5x = 6$ 3. $\ln(3x+1)^2 > 8$

B. Investments Compounded Continuously

Key Concept — **Continuously Compounded Interest** — For Your FOLDABLE

Calculate continuously compounded interest using the following formula.

$$A = Pe^{rt},$$

where A is the amount in the account after t years, P is the principal amount invested, and r is the annual interest rate.

Examples

1. Suppose you deposit \$700 into an account paying an APR of 3%, compounded continuously. What is the balance after 8 years?

2. Suppose you deposit \$700 into an account paying an APR of 3%, compounded continuously. How long will it take for the balance in your account to reach at least \$1200?

3. Suppose you deposit an unknown amount into an account paying an APR of 3%, compounded continuously. How much would have to be deposited in order to reach a balance of \$1500 after 12 years?

Homework: p. 529 – 1-19 all, 31, 40, 47, 68, 71, 73, 78

Section 8.8 – Using Exponential and Logarithmic Functions

Objectives:

1. To use logarithms to solve problems involving exponential growth and decay.
2. To use logarithms to solve problems involving logistic growth.

Continuous Exponential Growth and Decay

A. The Function

Key Concept **Exponential Growth and Decay** *For Your FOLDABLE*

Exponential Growth	Exponential Decay
Exponential growth can be modeled by the function $f(x) = ae^{kt}$, where a is the initial value, t is time in years, and k is a constant representing the **rate of continuous growth**.	Exponential decay can be modeled by the function $f(x) = ae^{-kt}$, where a is the initial value, t is time in years, and k is a constant representing the **rate of continuous decay**.

B. Examples

1. The half-life of Sodium-22 is 2.6 years.
   a) Determine the rate of decay for Sodium-22.

   b) A geologist examining a meteorite estimates that it contains only about 10% as much Sodium-22 as it would have contained when it reached the surface of the Earth. How long ago did the meteorite reach the surface of the Earth?

2. In 2007, the population of China was 1.32 billion. In 2000, it was 1.26 billion.
   a) Determine China's relative rate of growth.

   b) When will China's population reach 1.5 billion?

   c) India's population in 2007 was 1.13 billion and can be modeled by $y = 1.13e^{0.015t}$. Determine when India's population will surpass China's.

Homework: p. 537 – 1-3 all, 4, 5, 7, 10, 25, 26