Section 4.1A – Antiderivatives and Indefinite Integration

Objectives:

1. Write the general solution of a differential equation.
2. Use indefinite integral notation for antiderivatives.
3. Use basic integration rules to find antiderivatives.

I. Antiderivatives

A. Definition – F is <u>AN</u> antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

B. Example:

1. What is an antiderivative of the following

   a) $3x^2$

   b) $\cos x$

2. What is the antiderivative of a constant?

II. Notation for Antiderivatives

A. Antidifferentiation is called Integration

B. Solving $\frac{dy}{dx} = f(x)$

$$y = \int f(x)dx = F(x) + C$$

C. $\int f(x)dx$ is read as the antiderivative of f with respect to x. Thus dx serves to identify x as the variable of integration.

D. $\int F'(x)dx =$ (i.e., integration is the "inverse" of differentiation)

E. $\frac{d}{dx}\left[\int f(x)dx\right] =$ (i.e., differentiation is the "inverse" of integration)

III. Basic Integration Rules

A. Rules: See p. 250

B. Examples

1. $\int dx$

2. $\int x^5 dx$

3. $\int (2\sin x)dx$

4. $\int (x^2 - 2x + 5)dx$

5. $\int \left(3\sqrt{x}\right)dx$

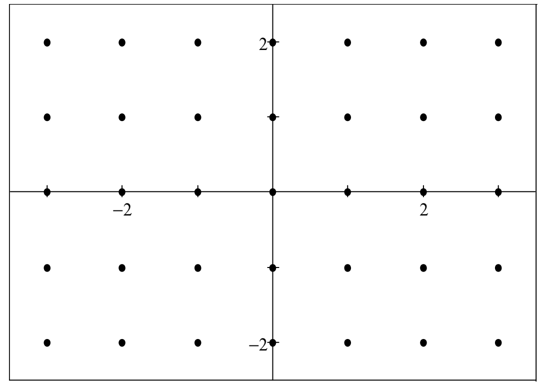
6. $\int \frac{x+1}{\sqrt{x}} dx$

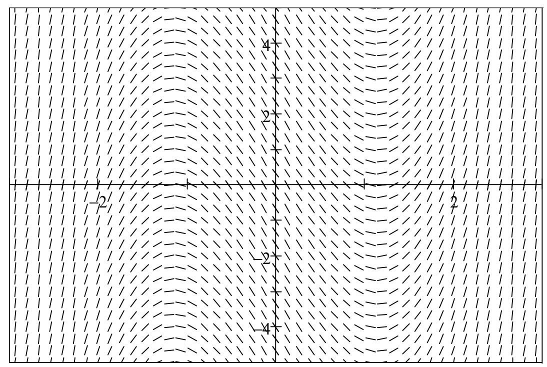
7. $\frac{d}{dx}\int (\tan x)dx$

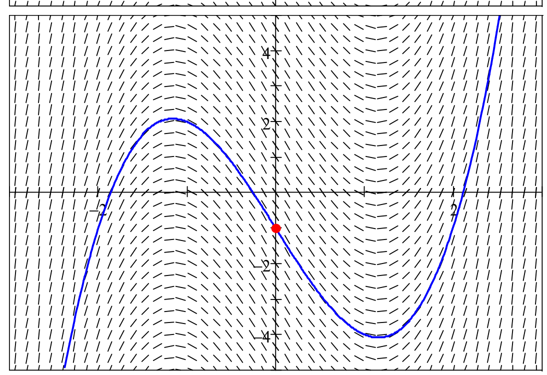
p. 248 – 5-8 all, 12, 11, 13, 21, 27, 29, 33, 34, 35, 37, 41

Section 4.1B – Antidifferentiation and Slope Fields

Objectives:
1. Find the families of antiderivatives using slope fields.
2. Find a particular solution of a differential equation.







I. Solution Curves and Slope Fields

A. What does $\dfrac{dy}{dx} = 2x$ represent?

1. Since we do not know what slopes we are dealing with we draw all the possible slopes or slope fields.

2. If we antidifferentiate how do we know which graph represents the original?

B. Questions:

1. What is a slope field?
A slope field is the graphical representation of a differential equation.

2. What Do Slope Fields Show You?
The solution curves are hiding in the slope field. Given one point of the particular solution curve, you can sketch the graph from that point, in both directions, to see the graph of the solution.

Do Calculator Lab #11

II. Initial Conditions and Particular Solutions

A. Examples:

1. Find the equation of the curve whose slope at a point (x, y) is $3x$, if the curve is required to pass through the point $(1, -1)$.

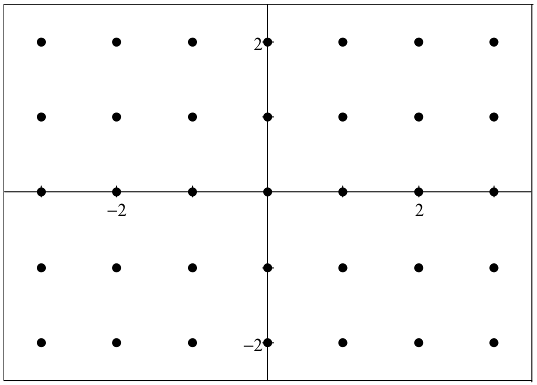
2. A heavy projectile is fired straight up from a platform 10 feet above the ground with an initial velocity of 160 ft/s. Assume that the only force affecting the projectile during its flight is gravity, which produces a downward acceleration of 32 ft/s^2. Find an equation for the projectile's height above the ground as a function of time, if $t = 0$ when the projectile is fired.

Homework:
Slope Field Worksheet and p. 248 – 49-54 all, 56, 59, 61, 64, 65, 70, 84, 87, 92-96 all

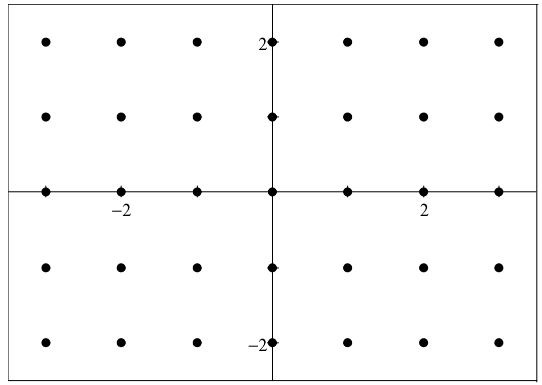
Worksheet – Slope Fileds

With out your graphing utilities or programs graph the slope field for the following differentials.

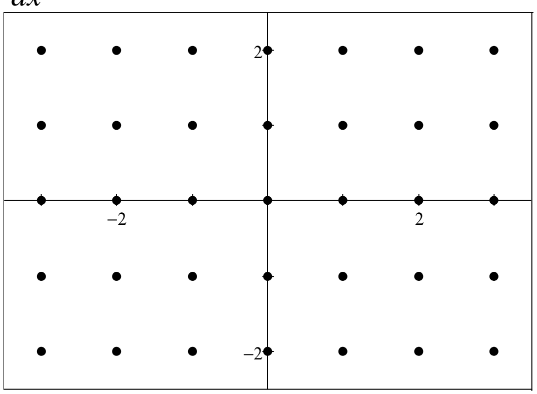
1. $\frac{dy}{dx} = x+1$



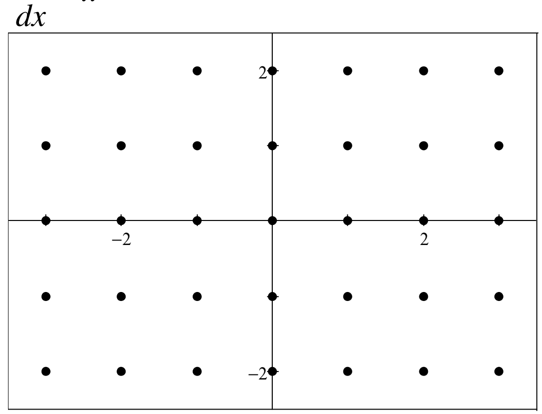
2. $\frac{dy}{dx} = 2y$



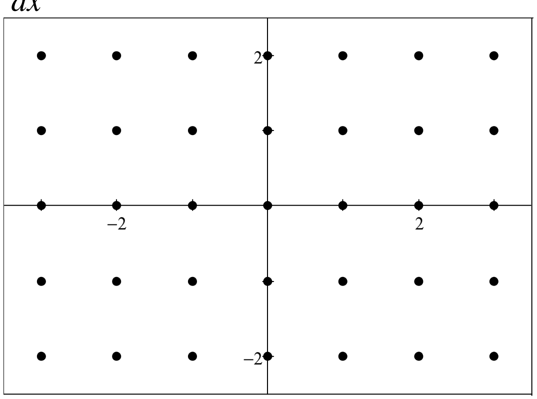
3. $\frac{dy}{dx} = x+y$



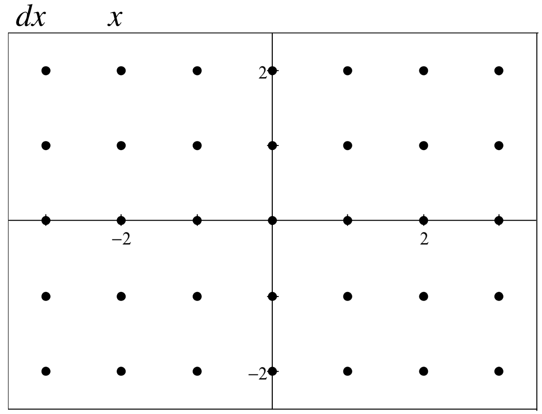
4. $\frac{dy}{dx} = x$



5. $\frac{dy}{dx} = y+1$



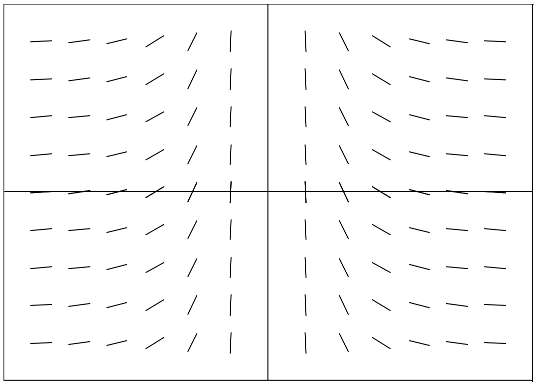
6. $\frac{dy}{dx} = -\frac{y}{x}$



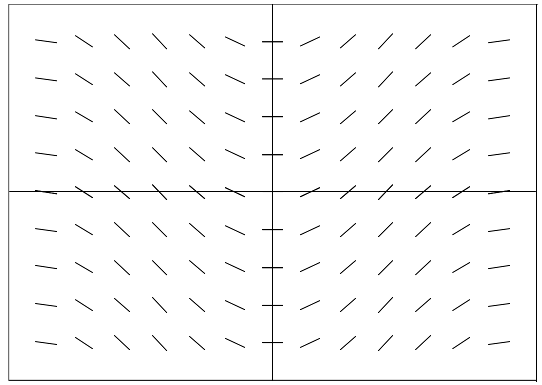
Worksheet – Slope Fileds (cont)

Match each slope field with the equation that the slope field could represent. (No calculators)

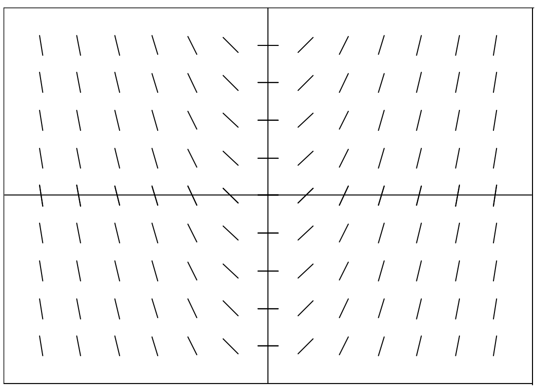
A.



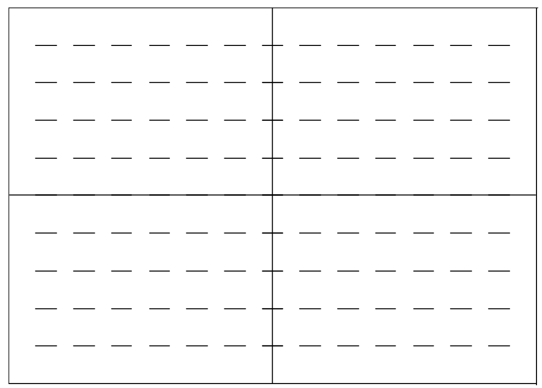
B.



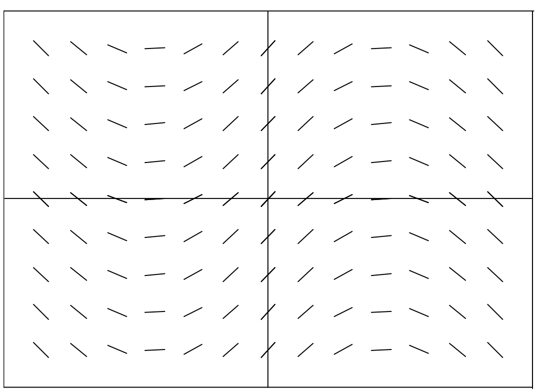
C.



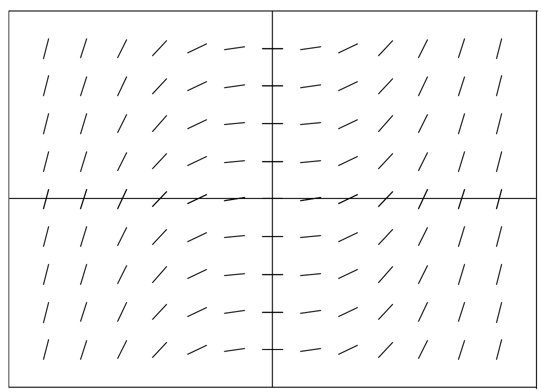
D.



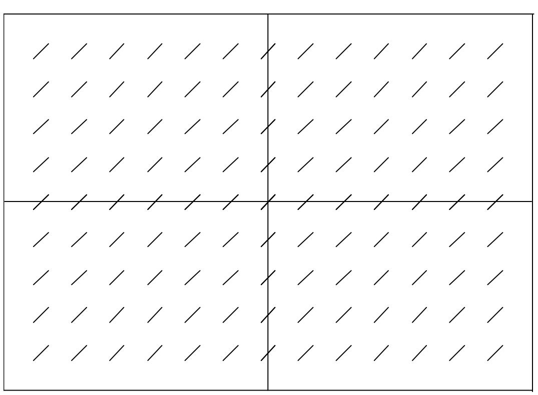
E.



F.



G.



10. $y = 1$

11. $y = x$

12. $y = x^2$

13. $y = \frac{1}{6}x^3$

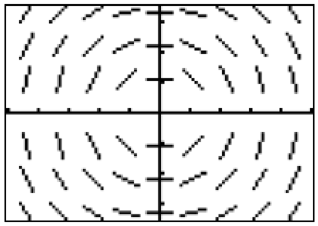
7. $y = \frac{1}{x^2}$

8. $y = \sin x$

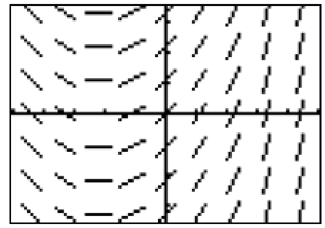
9. $y = \cos x$

Match the slope fields with their differential equations.

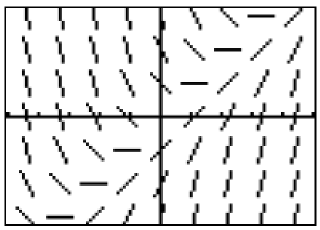
(A)



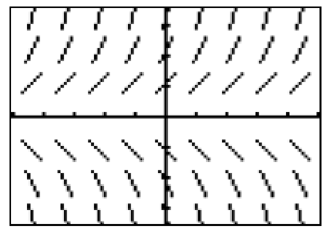
(B)



(C)



(D)



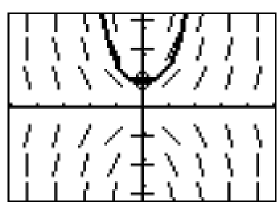
15. $\frac{dy}{dx} = \frac{1}{2}x + 1$

16. $\frac{dy}{dx} = y$

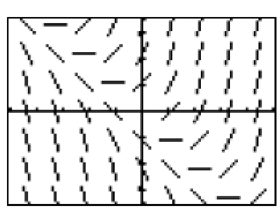
17. $\frac{dy}{dx} = x - y$

18. $\frac{dy}{dx} = -\frac{x}{y}$

19. The calculator drawn slope field for the differential equation $\frac{dy}{dx} = xy$ is shown in the figure below. The solution curve passing through the point (0, 1) is also shown.
   (a) Sketch the solution curve through the point (0, 2).
   (b) Sketch the solution curve through the point $(0, -1)$.

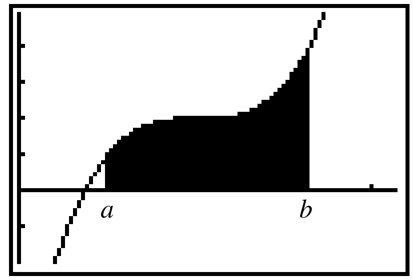


20. The calculator drawn slope field for the differential equation $\frac{dy}{dx} = x + y$ is shown in the figure below.
   (a) Sketch the solution curve through the point (0, 1).
   (b) Sketch the solution curve through the point $(-3, 0)$.



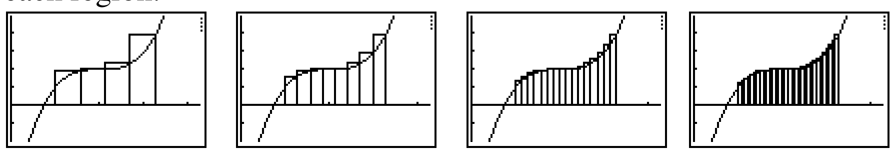
Objectives:
1. Understand the concept of area.
2. Approximate the area of a plane region.
3. Understand the definition of a Riemann sum.

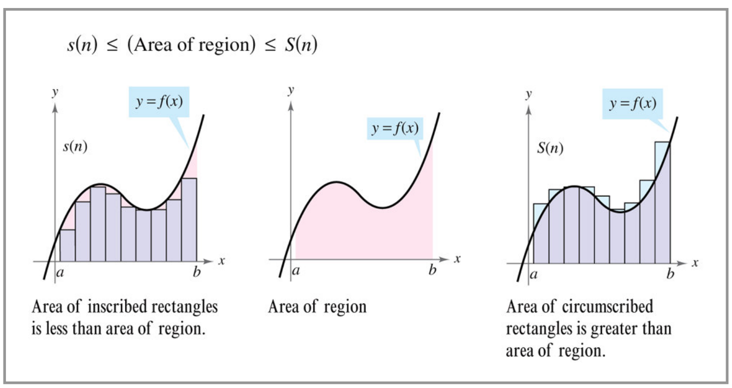
I. The Area of a Plane Region



A. Recall we there were two classic problems in calculus – the tangent problem and the area problem.
We have discussed the tangent problem so now we will begin our investigation of the area problem.

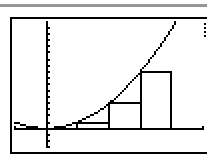
B. Method:
1. Break region into rectangles or trapezoids (of equal size, if possible).
2. Find the area of each region.
3. Find total area.



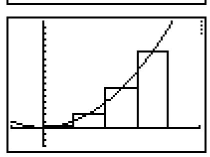


By increasing the number of rectangles you can obtain closer and closer approximations.

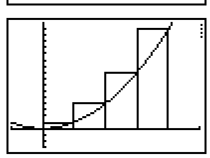
C. Four Types
Rectangular Methods
1. Left Sided Rectangles



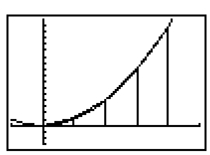
2. Middle Rectangles



3. Right Sided Rectangles



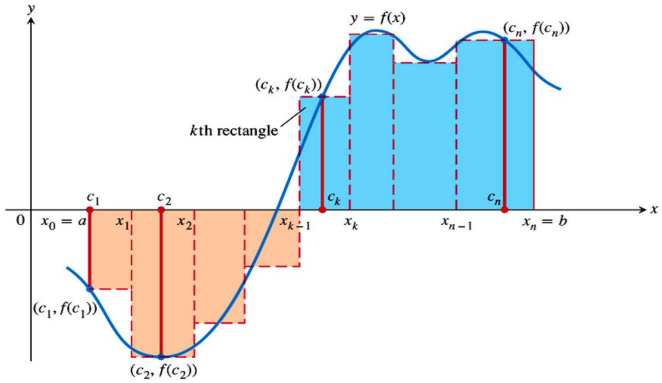
Trapezoid Method



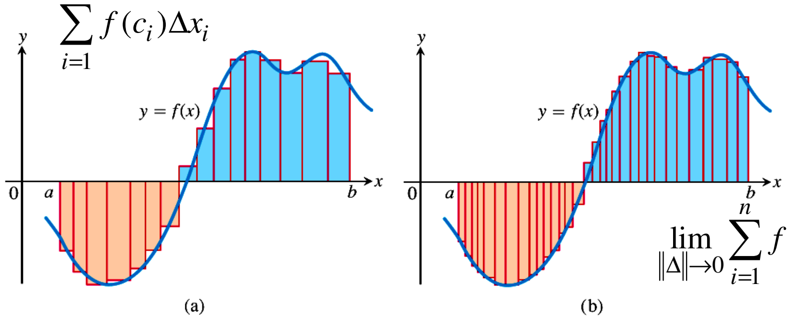
Do Calculator Lab #10

II. Riemann Sums

$$R_n = f(c_1)\Delta x_1 + f(c_2)\Delta x_2 + \ldots + f(c_n)\Delta x_n = \sum_{i=1}^{n} f(c_i)\Delta x_i$$



$$\sum_{i=1}^{n} f(c_i)\Delta x_i$$



$$\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i = L$$

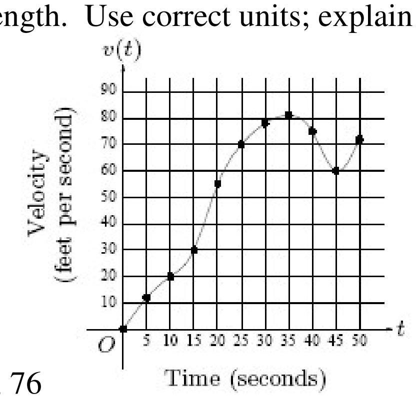
III. The Definite Integral

A. $I = \int_a^b f(x)dx = \lim_{\|\Delta\|\to 0} \sum_{k=1}^{n} f(c_i)\Delta x_i$

B. Definite Integral – If f is _______________ and _______________ on the ________ interval $[a,b]$, then the _____ of the region __________ by the graph of _____, the ________, and the vertical _______ $x = a$ and $x = b$ is given by

Area =

C. Example – Approximate $\int_0^{50} v(t)dt$ with a Riemann Sum, using the midpoints of five subintervals of equal length. Use correct units; explain the meaning of the integral.



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

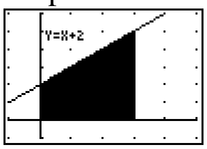
Homework:
p. 268 – 29, 31, 33, 35, 42, 43, 73, 76
p. 280 – 9-12, 53, 54; and p. 316 – 3, 7, 52 (Trapezoid Rule Only)

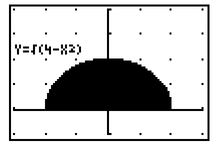
Section 4.3B – Definite Integrals

Objectives:
1. Express the area under a curve as a definite integral.
2. Evaluate a definite integral using properties of definite integrals.

I. Expressing Area as a Definite Integral.

Examples – express as a definite integral and find the area.

1. 

2. 

II. Properties of Definite Integrals

A. Properties

1. If *f* is defined at $x = a$, then $\int_a^a f(x)dx =$
2. If *f* is integrable on [*a*,*b*], then $\int_a^b f(x)dx =$
3. If *f* is integrable on the three closed intervals determined by *a*, *b*, and *c*, then
$\int_a^c f(x)dx =$
4. For f, g integrable on [a, b], and k is a constant ... ,
then since kf and $f \pm g$ are integrable on [a, b], we have :
a) $\int_a^b kf(x)dx =$
b) $\int_a^b [f(x) \pm g(x)]dx =$
5. If *f* and *g* are integrable on the closed interval [*a*,*b*] and $f(x) \le g(x)$ for every *x* in [*a*,*b*] , then $\int_a^b f(x)dx \le$

B. Examples

Given: $\int_{-1}^{1} f(x)dx = 5$, $\int_{1}^{4} f(x)dx = -2$, $\int_{-1}^{1} h(x)dx = 7$, and $g(x) \ge f(x)$ on $[-1,1]$.

1. $\int_4^1 f(x)dx =$

2. $\int_{-1}^{1} [2f(x) + 3h(x)]dx =$

3. $\int_{-1}^{1} [f(x) - h(x)]dx =$

4. $\int_{-1}^{4} f(x)dx =$

5. Represent as an inequality: $\int_{-1}^{1} g(x)$

Homework:
p. 280 – 13-22 all, 23, 27, 31, 33-43 odds, 47, 49, 52, 65, 66

Section 4.4 – The Fundamental Theorem of Calculus

Objectives:

1. Evaluate a definite integral using the Fundamental Theorem of Calculus.
2. Understand and use the Mean Value Theorem for Integrals.
3. Find the average value of a function over a closed interval.
4. Understand and use the Second Fundamental Theorem of Calculus.

I. The Fundamental Theorem

A. The Theorem:

If a function *f* is continuous on the closed interval $[a,b]$ and F is an antiderivative of *f* on the interval $[a,b]$, then $\int_a^b f(x)dx = .$

For instance: $\int_1^3 x^3 dx =$

B. Examples

1. $\int_{-1}^{3} \left(x^3 + 1\right)dx$

2. $\int_1^4 3\sqrt{x}dx$

3. $\int_0^{\pi/4} \left(\sec^2 x\right)dx$

4. Using FTC with absolute values $\int_0^2 |2x-1|\,dx$

5. Using FTC to find Area: Find the area of the region bounded by the graph $y = 2x^2 - 3x + 2$, the x-axis, and vertical lines $x = 0$ and $x = 2$.

II. The Mean Value Theorem (Average Value Theorem) for Integrals

A. Note: With __________ Sums you saw the _______ of a region under a _________ is greater than the __________ of an inscribed ____________ and less than the area of a circumscribed ________________. The Mean Value Theorem for Integrals state that somewhere "_________" the inscribed and circumscribed rectangles there is a ___________ rectangle whose area is precisely __________ to the area of the region under the curve.

B. Theorem: $(b-a)f(c) = \int_a^b f(x)dx$

C. Average Value Function: $f(c) =$

This means – If $f(x)$ is __________, then this will find the __________ velocity of the interval [a,b].

D. Examples

1. Find the average value of $f(x) = 4 - x^2$ on the interval [-2,2].

2. Find the c that will satisfy the Mean Value.

3. Suppose a wholesaler receives a shipment of 1200 cases of chocolate bars every 30 days. The chocolate is sold to retailers at a steady rate, and x days after the shipment arrives, the inventory of cases still on hand is $I(x) = 1200 - 40x$. Find the average daily holding cost if one case costs $0.03/day to hold.

III. The Second Fundamental Theorem of Calculus

A. $\frac{d}{dx}\left[\int_a^u f(t)dt\right] =$ as well as $\frac{d}{dx}\left[\int_v^u f(t)dt\right] =$

Example Proof: Find the derivative of $F(x) = \int_{\pi/2}^{x^3} (\cos t)dt$

B. Evaluate:

1. $\frac{d}{dx}\left[\int_1^{x^2} e^{3t-2}dt\right]$

2. $\frac{d}{dx}\left[\int_{-x}^{2x} 2t^3 dt\right]$

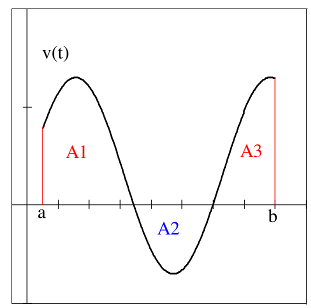
IV. Net Change Theorem

A. Theorem: The definite integral of the rate of change of a quantity *F'*(*x*) give the total change, or net change, in that quantity over the interval [*a*, *b*].

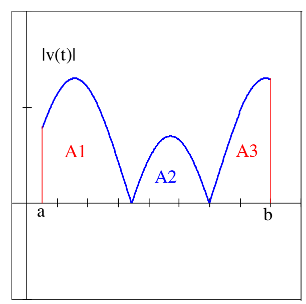
$$\int_a^b F'(x)\,dx = F(b) - F(b)$$

B. Example: A chemical flows into a storage tank at a rate of 180 + 3t liters per minute for the first 60 minutes. Find the amount of chemical that flows into the tank during the first 20 minutes.

C. Physics Concepts:



1. Displacement: $\int_a^b v(t)dt = A1 - A2 + A3$



2. Distance: $\int_a^b |v(t)|dt = A1 + A2 + A3$

3. Example: A particle moving along a line so that its velocity is $v(t) = t^3 - 10t^2 + 29t - 20$ feet per second at time t.
   a) What is the displacement of the particle on the time interval [1,5]?

   b) What is the distance traveled by the particle on the time interval [1,5]?

Homework:
p. 293 – 1-4 all, (6-30)/3, 36, 37, 40, 43, 46, 50, 51, 53, 54, 56, 57, 59, 60, 61, 63, 66, 74, 77, 81-91 odds, 94, 98, 101, 102, 103, 105

Section 4.5 – Integration by Substitution

Objectives:

1. To use the substitution method to compute integrals

I. Recognizing the Pattern

A. Antidifferentiation of a composite Function: $\int f(u)du = F(u)+C$

B. Examples

1. $\int \left[2x(x^2+1)^4\right]dx$

2. $\int \left(x^2\sqrt{x^3+1}\right)dx$

3. $\int \left[2\sec^2 x(\tan x+3)\right]dx$

Guidelines for Making a Change of Variables

1. Choose a substitution $u = g(x)$. Usually, it is best to choose the *inner* part of a composite function, such as a quantity raised to a power.
2. Compute $du = g'(x)\,dx$.
3. Rewrite the integral in terms of the variable u.
4. Find the resulting integral in terms of u.
5. Replace u by $g(x)$ to obtain an antiderivative in terms of x.
6. Check your answer by differentiating.

II. Integration by Substitution

A. Indefinite Integrals

1. $\int \cos(7x+5)dx$

2. $\int \frac{2xdx}{\sqrt[3]{x^2+1}}$

3. $\int \left(\sin^4 x\cos x\right)dx$

4. ** $\int \left(\tan^2 x\right)dx$

5. ** $\int \left(x\sqrt{2x-1}\right)dx$

6. ** $\int \frac{x}{\sqrt{2x-1}}dx$

B. Definite Integrals

1. $\int_0^{\pi/4} \left(\tan x\sec^2 x\right)dx$

2. $\int_{-1}^{1} \left(3x^2\sqrt{x^3+1}\right)dx$

Homework: p. 306 – 1-6 all, (12-27)/3, 43, 45, (48-84)/3, 89, 91, 93, 95, 97, 102, 123, 129, 130, 132