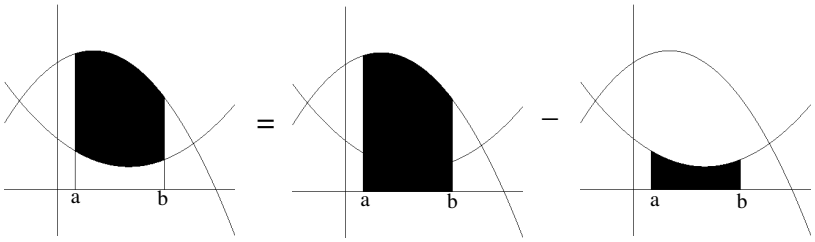
Section 6.1 – Area of Regions between two Curves

Goals:
1. To find the area between two curves

I. Area of a Region Between Two Curves

A. Graphical Representaion



B. Integral Representaion

$\int_a^b [f(x) - g(x)]dx =$

C. Example

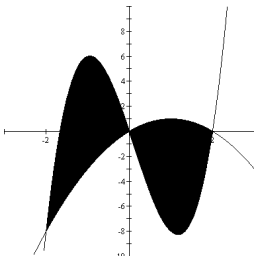
1. Find the area of the region bounded by the graphs of $y = x^2 + 2$, $y = -x$, $x = 0$, and $x = 1$.

2. Find the area between $y = \cos x$ and $y = \sin x$ from 0 to $\pi/2$.

II. Area of region between intersecting curves

A. Examples

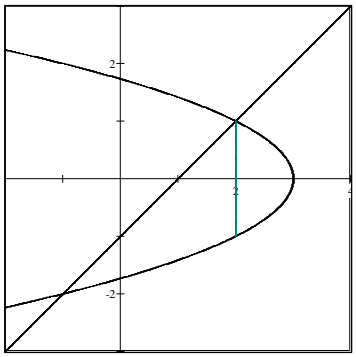
Find the area of the region between the graphs of $f(x) = 3x^2 - x^2 - 10x$ and $f(x) = -x^2 + 2x$.



B. Horizontal vs Vertical Representation

1. Example:

a) Find the area of the region bounded by $y = \pm\sqrt{3-x}$ and $y = x-1$ using vertical representation.

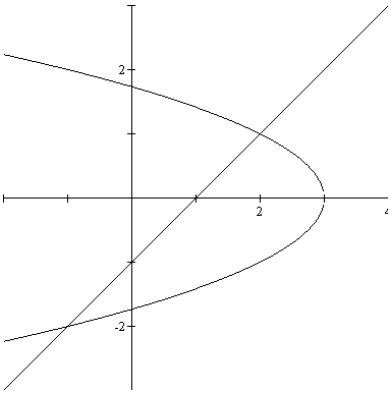


b) Find the same area above but using horizontal representation.

(1) Convert to $x =$

$x = 3 - y^2$ and $x = y + 1$

(2) Integrate in relation to y



2. Find the area between $y = \pm\sqrt{x}$, $y = x - 2$

Homework: p. 454 – 1-6 all, 17, 19, 23, 27, 31, 35, 41, 47, 49, 53, 59, 79, 80

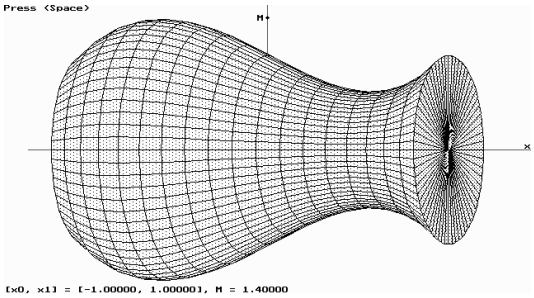
Section 6.2 – Volume the Disk Method

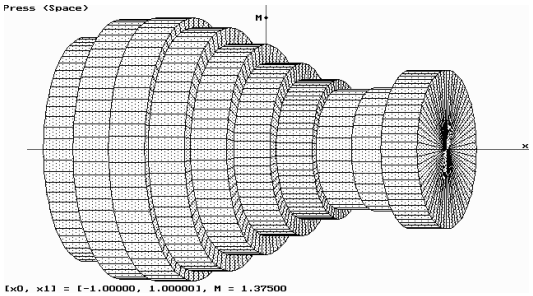
Goals:

1. To find the volume of a solid of revolution by using the disk method.
2. To find the volume of a solid of revolution by using the washer method.
3. To find the volume of a solid of revolution by using a known cross section.

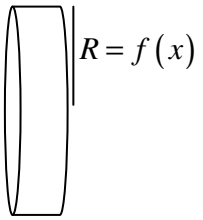
I. The Disk Method

A. Graphical Representation of a function rotated around a horizontal axis.





1. Volume of one disk of rotation about the *x*-axis



2. Volume of many disks rotated about the *x*-axis

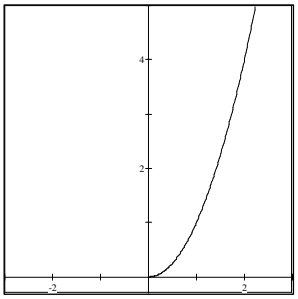
3. Examples

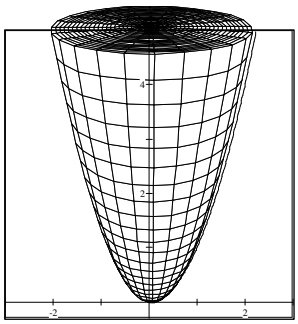
a) Find the volume of a region bound by $y = \sqrt{x}$, $0 \le x \le 4$, and the *x*-axis rotated about the *x*-axis.

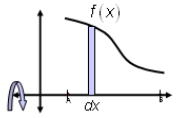
b) Find the volume of a region enclosed by the semicircle $x^2 + y^2 = 4$ and the *x*-axis rotated about the *x*-axis.

c) Find the volume generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1$ and $x = 4$ about the line $y = 1$.

B. Revolving about a vertical axis.



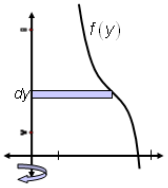




Horizontal Axis of revolution:

$$V = \pi \int_a^b \left[f(x) \right]^2 dx$$

radius width



Vertical Axis of revolution:

$$V = \pi \int_a^b \left[f(y) \right]^2 dy$$

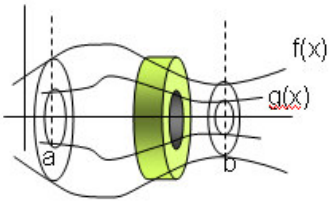
radius width

1. Example:

Find the volume of the region bounded by $y = \frac{1}{x^2}$, $1 \le y \le 4$, and the y-axis rotated about the y-axis.

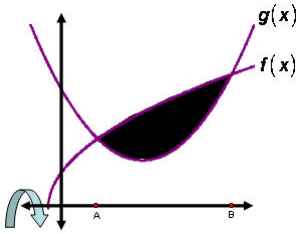
II. The Washer Method

Consider the area between two functions rotated about an axis of rotation



A. Graphical Representation

1. How do you find the volume of the figure formed by revolving the shaded area about the x-axis?



Examples

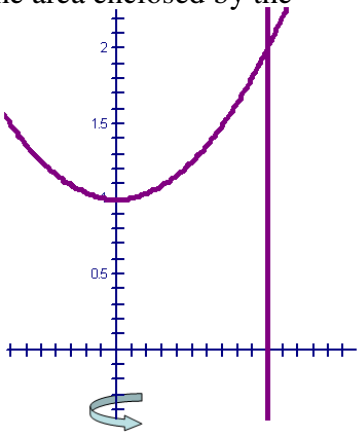
1. Find the volume of the region bounded by $y = x^2 + 1$ and $y = -x + 3$ revolved about the x-axis.

2. Find the volume of the region bounded by $y = x^2 + 1$ and $y = -x + 3$ revolved about the line $y = 5$

3. Find the volume of the region bounded by $y = x^2$ and $y = 2x$ revolved about the y-axis.

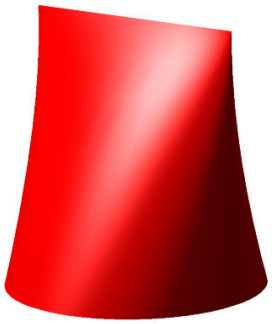
4. Find the volume of the region bounded by $y = x^2$ and $y = 2x$ revolved about the line $x = 2$.

5. Find the volume of the solid formed by revolving the area enclosed by the given functions about the y-axis.
$y = x^2 + 1,\ y = 0,\ x = 0,\ x = 1$



II. Find volume of a Solid with Known Cross Sections

A. Graphical Representation



B. Formulas

Example: Find the volume of a solid whose base is the region bounded by the lines $y = 1 - \frac{x}{2}$, $y = -1 + \frac{x}{2}$, and $x = 0$, with cross sections taken perpendicular to the *x*-axis are squares.

C. For cross sections of area $A(y)$ taken perpendicular to the y-axis,

Example: Find the volume of a solid whose base is bound by the graph $y = 4 - x^2$, y-axis, and the *x*-axis, with cross sections taken perpendicular to the *y*-axis are semicircles.

Homework:
Day 1: p. 465 – 1-4 all, 7-10 all, 23, 27, 31-36 all
Day 2: p. 465 – 5, 6, 11, 13, 20, 21, (42-48) evens,
Day 3: p. 468 – 71, 72