

Score: _____

Name: _____

Record all answers on answer sheet.

Section I – No calculators (Please show all work on separate paper)

Grading Scale
(37 questions total)

100% = 34	78% = 28
95% = 33	75% = 26
90% = 32	70% = 24
88% = 31	68% = 23
85% = 30	65% = 22
80% = 29	60% = 20
	55% = 19
	50% = 17
	45% = 15

1. If $f(x) = 5x^{1/3}$, then $f'(8) =$ _____

2. $\lim_{x \rightarrow \infty} \frac{5x^2 - 3x + 1}{4x^2 + 2x + 5} =$ _____

3. If $f(x) = \frac{3x^2 + x}{3x^2 - x}$ then $f'(x)$ is _____ (write as a single fraction)

4. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2 - 7x + 12}{x - 4}$ when $x \neq 4$, then $f(4) =$ _____

5. If $x^2 - 2xy + 3y^2 = 8$, then $\frac{dy}{dx} =$ _____ (write as a single fraction)

6. If $f(x) = \sin x + \csc x$, then $f'(x) =$ _____

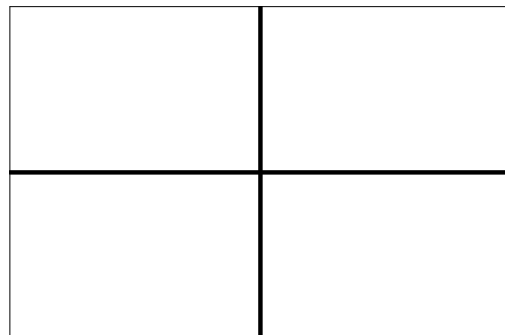
7. An equation of a line normal to the graph of $y = \sqrt{3x^2 + 2x}$ at $(2,4)$ is _____

8. If $f(x) = \cos^2 x$, then $f''(\pi) =$ _____

9. If $f(x) = \frac{5x}{x^2 + 1}$ and $g(x) = 3x - 2$, then $g(f(2)) =$ _____

10. Let f be the function given by $f(x) = 2x^4 - 4x^2 + 1$.

- Find an equation of the line tangent to the graph at $(2,17)$. _____
- Find the x - and y -coordinates of the relative maxima and relative minima. _____
- Find the x - and y -coordinates of the points of inflection. _____
- Sketch the graph of $f(x)$. Graph:



11. The equation $y = 2 - 3 \sin \frac{\pi}{4}(x-1)$ has a fundamental period of _____

12. For what value of x does the function $f(x) = x^3 - 9x^2 - 120x + 6$ have a local minimum? ____

13. If $f(x) = \begin{cases} x^2 + 5 & \text{if } x < 2 \\ 7x - 5 & \text{if } x \geq 2 \end{cases}$, for all real numbers x , which of the following must be true? Justify

- I. $f(x)$ is continuous everywhere.
- II. $f(x)$ is differentiable everywhere.
- III. $f(x)$ has a local minimum at $x = 2$.

- (A) I only (B) I and II only (C) II and III only
(D) I and III only (E) I, II, and III

Justify: _____

_____.

14. If $f(x) = \sqrt{(x^3 + 5x + 121)}(x^2 + x + 11)$ then $f'(0) =$ _____

15. $\lim_{x \rightarrow 0} 4 \frac{\sin x \cos x - \sin x}{x^2} =$ _____

16. If $f(x) = \cos^3(x+1)$, then $f'(\pi) =$ _____

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Section II (calculators may be used)

17. Consider the equation $x^2 - 2xy + 4y^2 = 52$. Exact values only.

- Write an expression for the slope of the curve at the point (x, y) . _____
- Find the equation(s) of the tangent lines to the curve at the point $x = 2$. _____
- Find $\frac{d^2y}{dx^2}$ at $(2, 4)$. _____

18. Water is draining at the rate of $48\pi \text{ ft}^3/\text{min}$ from a conical tank whose diameter at its base is 40 feet and whose height is 60 feet.

- Find an expression for the volume of water in the tank in terms of its radius. _____
- At what rate is the radius of the water in the tank shrinking when the radius is 16 ft? _____
- How fast is the height of the water in the tank dropping at the instant the radius is 16 ft? _____

19. Cars A and B leave the same place at the same time. Car A heads due North at 10 km/hr. Car B heads due East at 12 km/hr. After 2.5 hours, how fast is the distance between the cars increasing (in km/hr)? _____

20. $\lim_{h \rightarrow 0} \frac{\sin\left(\frac{\pi}{6} + h\right) - \sin\left(\frac{\pi}{6}\right)}{h} =$ _____

21. Find two nonnegative numbers x and y whose sum is 100 and for which xy^2 is a maximum.

22. A 20-foot ladder slides down a wall at 5 ft/sec. At what speed is the bottom sliding out when the top is 10 feet from the floor? (in ft/sec) _____

23. The graph of $y = x^3 - 5x^2 + 4x + 2$ has a local minimum point(s) at _____

24. Find the value(s) of $\frac{dy}{dx}$ of $x^2y + y^2 = 5$ at $y = 2$

25. The graph of $y = 5x^4 - x^5$ has an inflection point (or points) at _____

26. The graph of $y = x^3 - 2x^2 - 5x + 2$ has local maximum at _____

27. If $f(x) = \left(1 + \frac{x}{20}\right)^5$, find $f''(40)$. _____

28. If the function $f(x)$ is continuous and differentiable $f(x) = \begin{cases} ax^3 - 6x; & \text{if } x \leq 2 \\ bx^2 + 4; & \text{if } x > 2 \end{cases}$ then $b =$ _____

29. Two particles leave the origin at the same time and move along the y -axis and their respective position determined by the functions $y_1 = \cos 3t$ and $y_2 = 5 \sin t$ for $0 < t < 2$. For what values of t , if any, do the particles have the same acceleration? _____

30. The temperature on New Years Day in Hinterland was given by $T(H) = -A - B \cos\left(\frac{\pi H}{12}\right)$, where T is the temperature in degrees Fahrenheit and H is the numbers of hours from midnight ($0 \leq H \leq 24$). If the initial temperature at midnight was $-15^\circ F$, and at Noon of New Year's Day was $5^\circ F$. Find an expression for the rate the temperature is changing with respect to H . _____