AP Worksheet #6 (Due: _____)

End of Chapter 4

Score: _____ Name: _____

Section I – No calculators (Please show all work)

Grading Scale
(60 Points Possible)

100%	= 56 correct	68%	= 38 correct
95%	= 54 correct	65%	= 36 correct
90%	= 52 correct	60%	= 34 correct
88%	= 50 correct	58%	= 32 correct
85%	= 48 correct	55%	= 30 correct
80%	= 46 correct	50%	= 28 correct
78%	= 44 correct	48%	= 26 correct
75%	= 42 correct	45%	= 24 correct
70%	= 40 correct	40%	= 22 correct

1. If $f(x)=2x^{1/4}$, then $f^{-1}(8)=$ _____

2. $\lim\limits_{x\to\infty}\dfrac{2x^2-3x+1}{3x^3+2x+5}=$ _____

3. If $f(x)=\dfrac{2x+1}{x-1}$ then $f'(x)$ is _____
(write as a single fraction)

4. If the function f is continuous for all real numbers and if $f(x)=\dfrac{x^2+x-12}{x+4}$ when $x\neq -4$, then $f(-4)=$ _____

5. If $x^3+4x^2y-3y^2=8$, then $\dfrac{dy}{dx}=$ _____ (write as a single fraction)

6. If $f(x)=\tan x+\sec^2 x$, then $f'(x)=$ _____

7. An equation of a line normal to the graph of $y=3x^2+2x-1$ at (2,15) is _____

8. $\int_{-1}^{1}\dfrac{4x}{(1+x^2)^2}\,dx=$ _____

9. If $f(x)=\tan^2 x$, then $f''(\pi)=$ _____

10. If $f(x)=\dfrac{5}{x^2+1}$ and $g(x)=3x$, then $f(g(2))=$ _____

11. $\int x\sqrt{5x^2-4}\,dx=$ _____ (write using rational exponents)

12. The slope of the line tangent to the graph $3x^2+5y^2=17$ at (2,1) is _____

13. The equation $y=1+5\sin\dfrac{\pi}{6}(x+5)$ has a fundamental period of _____

14. For what value of x does the function $f(x)=2x^3-18x^2-240x$ have a local minimum? ___

15. If $f(x)=\begin{cases}x^2+5 \text{ if } x<2\\ 4x-5 \text{ if } x\geq 2\end{cases}$, for all real numbers x, which of the following must be true? Justify.

I. $f(x)$ is continuous everywhere.
II. $f(x)$ is differentiable everywhere.
III. $f(x)$ has a local minimum at $x=2$.

(A) I only (B) II only (C) II and III only
(D) III only (E) I, II, and III

_____
_____
_____
_____.

16. The acceleration of a particle moving along the *y*-axis at time *t* is given by $a(t) = 4t - 12$. If the velocity is 10 when $t = 0$ and the position is 4 when $t = 0$, then the particle is changing direction at $t =$ _____

17. The average value of a function $f(x) = (x-1)^2$ on the interval from $x = 1$ to $x = 5$ is _____

18. If $F(x) = \int \sqrt{(x^3 + 3x + 121)}(x^2 + 1)dx$ then $F(x) =$ _____

19. $\lim_{x \to 0} \dfrac{\sin 2x \cos x - \sin 2x}{x^2} =$ _____

20. If $f(x) = \tan^3(x + \pi)$, then $f'(\pi) =$ _____

21. $\int x\sqrt{x+3}\ dx =$ _____

22. $\dfrac{d}{dx}\left[\int_2^{x^2} \ln(3t-5)dt\right] =$ _____

23. If a particle moves on a line according to the law $s = t^5 + 2t^3$, then how many times does it reverse directions? _____

24. A rectangular pigpen is to be built against a wall so that only three sides will require fencing. If *p* feet of fencing are to be used, the area of the largest possible pen is _____.

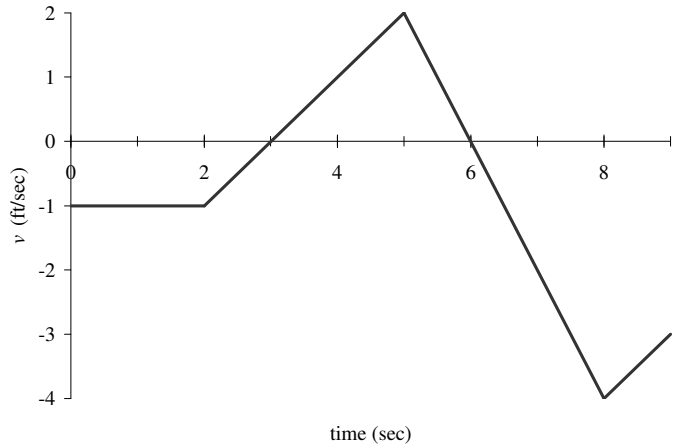
25. A smooth curve with equation $y = f(x)$ is such that its slope at each *x* equals x^2. If the curve goes through the point (-1,2), then its equation is _____.

26. If $G(2) = 5$ and $G'(x) = \dfrac{10x}{9 - x^2}$, then an estimate of *G* (2.2) using local linearization is approximately _____. (Refer to page 228 in your text book.)

27. The average value of $f(x) = 3 + |x|$ on the interval [-2,4] is _____.

28. Suppose $f(x) = \dfrac{x^2 + x}{x}$, if $x \neq 0$ and $f(0) = 1$. Prove below that *f* is continuous at $x = 0$.

Section II (calculators may be used)



The graph shown is for questions 29 and 30. It shows the velocity of an object during the interval $0 \le t \le 9$.

29. The object obtains the greatest speed at $t =$ _____.

30. The object's position was at the origin at $t = 3$. It returned to the origin at _____.

31. $\int_0^{\pi/4} \sin x \, dx + \int_{-\pi/4}^{0} \cos x \, dx =$ _____

32. $\lim_{h \to 0} \dfrac{\sec\left(\dfrac{\pi}{6} + h\right) - \sec\left(\dfrac{\pi}{6}\right)}{h} =$ _____

33. If $\int_{30}^{100} f(x)dx = A$ and $\int_{50}^{100} f(x)dx = B$, then $\int_{30}^{50} f(x)dx =$ _____

34. If $f(x) = 3x^2 - x$, and $g(x) = x^2$, then $\int g(f(x))dx =$ _____

35. The graph of $y = 2x^3 - 5x^2 + x + 2$ has a local minimum at _____

36. The average value of the function $f(x) = \dfrac{2x^2 - 3x + 1}{x - 1}$ on the interval [2,4] is _____

37. $\dfrac{d}{dx}\left(\int_0^{3x} \cos(t)dt\right) =$ _____

38. If the definite integral $\int_1^3 (x^2 + 1)dx$ is approximated by using the Trapezoid Rule when $n = 4$, the error from the actual is _____.

39. $\int \left(\cot^2 x\right) dx =$ _____

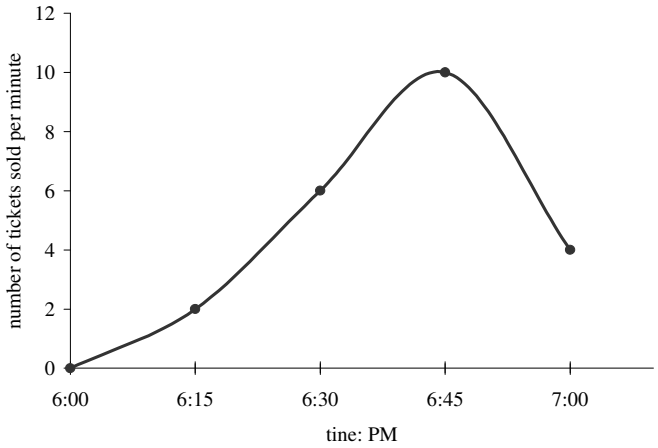
40. Find the distance traveled (to three decimal places) in the first 4 seconds, for a particle whose velocity is given by $v(t) = 7\sin^2 t$; where *t* stands for time. _____

41. $\int \tan^6 x \sec^2 x \, dx =$ _____

42. The intervals on which the function $f(x) = x^4 - 4x^3 + 4x^2 + 6$ increases are (is) _____.

43. If we replace $\sqrt{x-2}$ by *u*, then $\int_3^6 \frac{\sqrt{x-2}}{x} dx$ is equivalent to the integral _____ (make sure the integral is in terms of *u*)

44. How many point of inflection does the function *f* have on the interval $0 \le x \le 6$ if $f''(x) = 2 - 3\sqrt{x}\left(\cos^3 x\right)$? _____



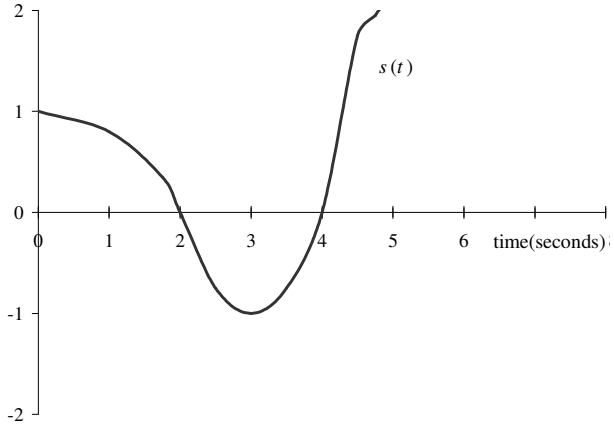
45. The graph shows the rate at which tickets were sold at a movie theater during the last hour before show time. Using right-rectangle method, and estimate of the size of the audience is _____.

Section III Free Response Questions (No calculator) – Work is to be shown on this page.
Note: On the free response sections I will be grading your written reasons as well as organization and neatness.

1) Let f be the function given by $f(x) = 1 + \frac{2}{x} + \frac{1}{x^2}$.
   a) Find the x and y intercepts. Justify.
   b) Write an equation for each vertical and horizontal asymptote for the graph of f. Justify.
   c) Find the intervals on which f is increasing and decreasing. Justify.
   d) Find the maximum and minimum values of f. Justify.

No Calculator – Work is to be shown on this page.

2) Let the graph of $s(t)$, the position function (in feet) of a moving particle, be given below. Let t be measured in seconds. The concavity changes at $t = 2$ and $t = 4$



a) Find the values of t for which the particle is moving to the right and when it is moving to left (i.e., when velocity is positive or negative, respectively). Justify.

b) Find the values of t for which the acceleration is positive and for which it is negative. Justify.

c) Find the values of t for which the particle is speeding up (i.e., when $|v|$ is increasing). Justify.

Section IV Free Response (calculator may be used) – Work is to be shown on this page.
Note: On the free response sections I will be grading your written reasons as well as organization and neatness.

3) A particle moves along the x-axis so that its acceleration at any time $t > 0$ is given by $a(t) = 12t - 18$. At time $t = 1$, the velocity of the particle is $v(1) = 0$ and the position $x(1) = 9$.
   a) Write an expression for the velocity of the particle $v(t)$.
   b) At what values of t does the particle change direction? Justify.
   c) Write an expression for the position function, $x(t)$, of the particle.
   d) Find the total distance traveled by the particle from $t = \frac{3}{2}$ to $t = 4$.

Calculator Allowed – Work is to be shown on this page.

4) A floodlight is on the ground 45 meters from a building. A thief 2 meters tall runs from the floodlight towards the building at 6 meters/second.
   a) Using a triangle(s) draw a picture of the situation.
   b) What is the relationship (equation) between the shadow on the building and the distance the thief is from the floodlight?
   c) How rapidly is the length of the shadow on the building changing when he is 15 meters from the building?