Section 14.1 – Basic Trig Identities

Objectives:
1. Use trigonometric identities to find trigonometric values.
2. Use trigonometric identities to simplify expressions.

I. Reciprocal Identities
Identities

1. $\sin A = \frac{1}{\quad}$ or $\csc A = \frac{1}{\quad}$ or $\_\_\_\_\_ = 1$

Note: $\frac{1}{\sin A} \neq \sin^{-1} A$

2. $\cos A = \frac{1}{\quad}$ or $\sec A = \frac{1}{\quad}$ or $\_\_\_\_\_ = 1$

3. $\tan A = \frac{1}{\quad}$ or $\cot A = \frac{1}{\quad}$ or $\_\_\_\_\_ = 1$

II. Quotient Identities
Identities

1. $\tan A =$ ——— Proof:

2. $\cot A =$ ———

III. Pythagorean Identities
Identities

1.
Proof:

2.

3.

IV. Examples:

1. Find $\tan\theta$ if $\sec\theta = -2$; $180° < \theta < 270°$

2. Find $\cos\theta$ if $\sin\theta = \frac{1}{3}$; $0° < \theta < 90°$

3. Simplify: $\sin\theta(\csc\theta - \sin\theta)$

4. Simplify: $\tan\theta \sin\theta \sec\theta$

5. The amount of light that a source provides to a surface is called the *illuminance*. The illuminance E in foot candles on a surface is related to the distance R in feet from the light source. The formula $\sec\theta = \frac{I}{ER^2}$ where I is the intensity of the light source measured in candles and θ is the angle between the light beam and a line perpendicular to the surface, can be used in situations in which lighting is important, as in photography. Solve the illuminance formula in terms of R.

6. Is the equation $R = \sqrt{\frac{I\cos\theta}{E}}$ equivalent to the equation $\frac{1}{R^2} = \frac{E}{I\sec\theta}$?

7. Solve the formula $A\cot\theta = \frac{F}{D^2}$ in terms of D.

8. Show $D = \sqrt{\frac{F\tan\theta}{A}}$ is equivalent to $F = \frac{AD^2\cos\theta}{\sin\theta}$.

Homework: p.894 – 1-8 all, (9-18)/3, 21, 23, 24, 26-28, 30, 32

Section 14.2 – Verifying Trig Identities

Objectives:
1. To use the basic trig identities to verify other identities.

A. Method
   1. _____: Transform the more _____ side of the _____ into the _____ of the _____ side.
   2. _____: _____ one or more basic trig _____ to simplify the expression.
   3. _____: _____ or _____ to simplify the expression.
   4. Always repeat the _____ and _____ step until goal is met.

   Note: When trying to verify possible trig identities, you can only work with one side. If you perform an operation with both sides, then you are assuming they are equal

B. Examples
   1. A. $\csc\theta\cos\theta\tan\theta = 1$ B. $\csc^2\theta\tan^2\theta\cos^2\theta = 1$

   2. $\dfrac{1+\cot x}{\cos x} = \csc x + \sec x$

   3. $\dfrac{\tan x - \sin x\cos x}{\sin^2 x} \equiv \tan x$

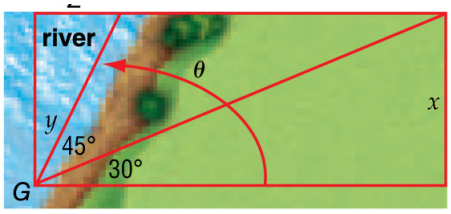
Homework: p.900 – (3-15)/3, (21-30)/3, 64-68 all, 71

Section 14.5 – Sum and Difference Identities

Objectives:
1. To use the sum and difference identities to find exact values and prove identities.

A. Identities
   1. $\cos(A \pm B) =$
   2. $\sin(A \pm B) =$
   3. $\tan(A \pm B) =$ ———————

B. Examples
   1. Find the exact value for $\sin 75°$ and $\cos(-75°)$
   2. Find the exact value for $\sin 105°$ and $\cos(-195°)$
   3. Find the dimensions of the lot.



   4. Verify: $\cos(360° - \theta) = \cos\theta$
   5. Verify: $\cos(\pi - \theta) = -\cos\theta$

Homework: p.906 – 1-6 all, 8-11 all, 14, 17, 22, 47, 51, 52

Section 14.6 – Double Angle and Half-Angle Identities

Objectives:
1. To use the double angle and half-angle identities to find exact values and prove identities.

A. Identities
   1. Double Angle Identities
      a) $\sin 2A =$ Proof:

      b) $\cos 2A =$ _____ or _____ or _____

      c) $\tan 2A =$ _____

   2. Half-Angle Identities
      a) $\sin \frac{A}{2} = \pm\sqrt{\frac{\quad}{2}}$ Proof:

      b) $\cos \frac{A}{2} = \pm\sqrt{\frac{\quad}{2}}$

      c) $\tan \frac{A}{2} = \pm\sqrt{\quad}$, $\cos A \neq -1$

B. Examples
   1. If $\sin x = -\frac{3}{4}$ and x terminates in quadrant IV. Find the exact values for:
      a) $\sin 2x$
      b) $\cos \frac{1}{2}x$
   2. Prove: $1 - \cos 2x \sec^2 x \equiv \tan^2 x$

   3. Use the half-angle identity to find the exact value for $\sin 15°$

Homework: p.794 – 13-37 odds, 48-57 all

Section 14.7 – Solving Trig Equations

Objectives:
1. To solve trig equations.

A. Examples

1. Solve $2\sin^2 x + \sin x - 1 = 0$ for the principle values of x.

2. Solve $\sin 2x = -2$ for all values of x.

3. Solve $4\cos x \sin x - 2\sin x - 2\sqrt{3}\cos x + \sqrt{3} = 0$ for all values of x.

4. Solve $1 - \cos 2x - \sin x = 0$ for $0 \le x \le 2\pi$.

5. Solve using a calculator: $\sin 3x - \cos 2x = 0$, $-2\pi \le x \le 4\pi$

Homework: p.802 – (15-39)/3, 47-55 all