Section 5.0A – Factoring Part 1

I. Work Together

A. Multiply the following binomials into trinomials. (Write the final result in descending order, i.e., $ax^2 + bx + c$).

$(x-7)(x+5)$

$(x+7)(x+2)$

$(2x+7)(3x+5)$

$(3x-8)(2x+7)$

B. Where did the first term (ax^2) of the trinomial come from in each of the problems above? (Use the words like product, sum, First terms, Outer terms, Inner terms, and Last terms)

C. Where did the last term (c) of the trinomial come from in each of the problems above? (Use the words like product, sum, First terms, Outer terms, Inner terms, and Last terms)

D. Where did the middle term (bx) of the trinomial come from in each of the problems above? (Use the words like product, sum, First terms, Outer terms, Inner terms, and Last terms)

E. Wrap Up

II. Think and Discuss

A. Factoring – Reversing the FOILing process.

1. Factor the following trinomials (i.e., break them apart to be two binomials again)

1. $x^2 + 10x + 9 = (\quad\quad)(\quad\quad)$

2. $x^2 - 3x - 10 = (\quad\quad)(\quad\quad)$

3. $x^2 - 8x + 16 = (\quad\quad)(\quad\quad)$

4. $4x^2 + x - 3 = (\quad\quad)(\quad\quad)$

5. $6x^2 - 11x - 2 = (\quad\quad)(\quad\quad)$

> Remember what you just discovered and what we discussed on the previous page (i.e., start with the first two questions and then verify with the last question).

B. Factoring can also be the reverse of distributing (i.e., what is in common with all the terms?)

1. Try the following

1. $15x^2y - 10xy^2 =$ ____$(\quad\quad\quad\quad)$

2. $x^2 + xy + 3x =$ ____$(\quad\quad\quad\quad)$

> Remember what you do when you distribute $3x(x - y + 2)$, now do the opposite.

C. What happens when both are together (reverse distribution and reverse FOILing)? Which do you do first? Always do reverse distribution first, then the reverse FOILing.

1. Try the following

1. $r^3 + 3r^2 - 54r =$ ___$(\quad\quad\quad\quad)$= ___$(\quad\quad)(\quad\quad)$

2. $2r^3 + 32r^2 + 128r =$ ___$(\quad\quad\quad\quad)$= ___$(\quad\quad)(\quad\quad)$

3. $4r^4 - 16r^2 =$ ___$(\quad\quad)$= ___$(\quad\quad)(\quad\quad)$

Homework: p. P8 – 1 – 24 all

Section 5.0B – Factoring Part 2

Objective:

1. Factoring polynomials with 4 or more terms.

I. Work Together

A. Multiply the following binomials.

1. $(x-a)(y+b)$

2. $(x+7)(y+2)$

3. $(x^2-2)(x+3)$

4. $(2x^2+7)(3x+5)$

B. Looking at the binomials you just multiplied, can you figure out a way to factor the following?

1. $ax+bx+ay+by=(\quad\quad)(\quad\quad)$

2. $14+7y+2x+xy=(\quad\quad)(\quad\quad)$

3. $x^3-3x^2+4x-12=(\quad\quad)(\quad\quad)$

4. $4x^3-6x^2+10x-15=(\quad\quad)(\quad\quad)$

C. Wrap Up

II. Think and Discuss

A. Factor the following using the grouping method.

1. $2x+10+ax+5a=($ $)+($ $)=$___$($ $)+$____$($ $)=($ $)($ $)$

2. $rs+st+3r+3t=($ $)+($ $)=$___$($ $)+$____$($ $)=($ $)($ $)$

3. $5xy-10x-2y+4=($ $)+($ $)=$___$($ $)+$____$($ $)=($ $)($ $)$

4. $10x^3-20x^2-2a+4=($ $)+($ $)=$___$($ $)+$____$($ $)=($ $)($ $)$

5. $5x^3-20x^2+3x-12=($ $)+($ $)=$___$($ $)+$____$($ $)=($ $)($ $)$

B. What happens if none of the methods I use is able to factor the problem? Then the polynomial is prime.

Try the following

1. x^2-36

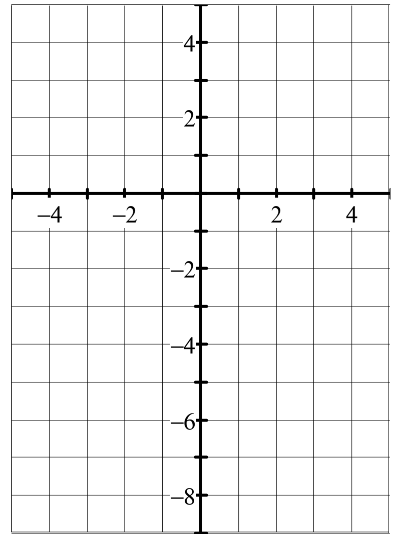
2. x^2-3x+4

3. x^2+9

4. $3x^3-2x^2y$

Homework: p. 272 – 20-37 all and p 274 – 71-78 all

Section 5.1 – Graphing Quadratics



Objectives:
1. Graph Quadratic Functions
2. Find the axis of symmetry and coordinates of the vertex of a parabola.
3. Model data using a quadratic function.

$y = 2x^2 - 4x - 5$

I. Think and Discuss
   A. Quadratic Functions
      1. Form –
         a) Quadratic term –
         b) Linear Term –
         c) Constant Term –
      2. Graph of a quadratic
         a) Symmetrical
            (1) Def –
            (2) Axis of Symmetry
               (a) Def –

               (b) Equation for a parabola:
         b) Vertex
            (1)
            (2)
            (3)

         c) Direction of Opening –
            (1) If *a* is positive, it opens ____________
            (2) If *a* is negative, it opens ____________

   B. Which points are on the graph of the function $y = 2x^2 - 4x - 5$?
      1. (1, -7)
      2. (2, 0)
      3. (0, -5)
      4. (-2, 11)
   C. What do you think the greatest exponent is for a quadratic function? for a linear function?

   D. Try This

      1. Tell whether each function is linear or quadratic?
         a). $y = (x-3)(x-2)$ b). $y = x(x+3)$
         c). $y = (x^2 + 5x) - x^2$ d). $y = (x-5)^2$

      2. Find the axis of symmetry, vertex, and determine if the vertex is a maximum or minimum point for the following: $y = 5 + 16x - 2x^2$

      3. Use you calculator to verify your answers to problem 2.

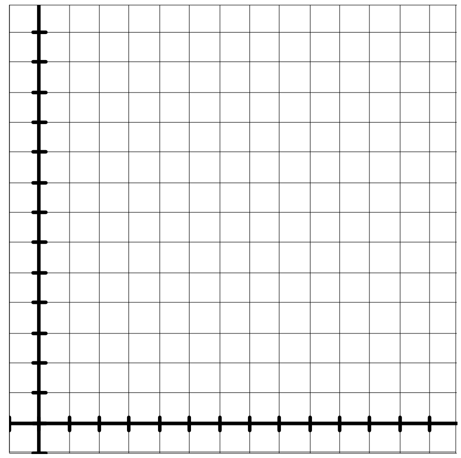
Homework: p. 254 – 1-11 all, 19, 27, 43-48 all, 61

Section 5.1B – Modeling Real World Data

Modeling Data

A. The table shows the average temperature in Gatlinburg, TN, for each month. Plot the points on a graph. Would it be useful to represent this data with a linear model? Explain.

Month	**Temp**
Feb(2)	52
Apr(4)	72
Jun(6)	84
Aug(8)	86
Oct(10)	71
Nov(11)	52



B. Finding Equations to model Quadratic Functions

1. Find an equation to model the data mentioned, using your graphing calculator. (Hint: It is done the same way you did the linear functions, except for one thing.)

2. Use the equation you just found to predict the average temperature in September.

3. How close was it to the actual temp of 81?

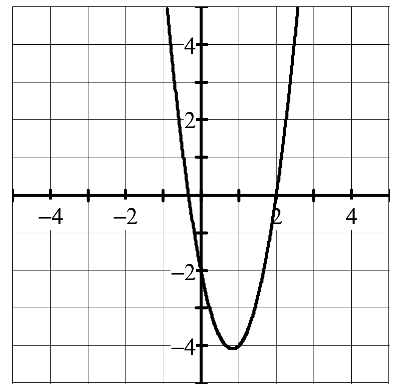
Homework: p. 258 – 1-4 all

Section 5.2 – Solving Quadratic Equations by Graphing

Objectives:

1. Solve quadratics by graphing

I. Solving Quadratic Functions – Graphically
   A. Solution (Root or Zero)
      1. Algebraic Definition –
      2. Graphical definition –
      (What would happen if the graph didn't cross the *x*-axis or just touched it?)



   B. Solve the given graph: $y = 3x^2 - 5x - 2$.

   C. Solve the following using your graphing calculator.
      1. $y = x^2 - 3x + 2$
      2. $y = 3x^2 + 12x + 12$

Homework: p. 263 – 1-13 all, 30-32 all, 49, 52 and p. 267 – 9-12 all

Section 5.3 – Solving Quadratics by Factoring

Objective:

1. Solve polynomials

I. Work Together

A. Solve for each variable in the following equations.

$3x = 0$

$6y = 0$

$5xy = 0$

–Why were these easy to solve?

B. Solve for each variable in the following equations.

$ab = 12$

$5xy = 75$

–Can I use the same method I did with the problems in section A above? Explain

C. Wrap Up

II. Think and Discuss

A. The Process

1.

2.

3.

4.

B. Try the following

$x^2+10x+9=0$

$(\quad)(\quad)=0$

$(\quad)=0$ *or* $(\quad)=0$

$x=$____ *or* $x=$_____

$x^2-3x-10=0$

$(\quad)(\quad)=0$

$(\quad)=0$ *or* $(\quad)=0$

$x=$____ *or* $x=$_____

$4x^2+x-3=0$

$(\quad)(\quad)=0$

$(\quad)=0$ *or* $(\quad)=0$

$x=$____ *or* $x=$_____

$12x^2-22x-4=0$

____$(\quad)(\quad)=0$

$(\quad)=0$ *or* $(\quad)=0$

$x=$____ *or* $x=$_____

$6x^3+x^2-2x=0$

___$(\quad)(\quad)=0$

___ $=0$ *or* $(\quad)=0$ *or* $(\quad)=0$

$x=$____ *or* $x=$_____ *or* $x=$_____

$4x^4-16x^2=0$

____$(\quad)(\quad)=0$

_____ $=0$ *or* $(\quad)=0$ *or* $(\quad)=0$

$x=$____ *or* $x=$_____ *or* $x=$_____

C. What is wrong with the following problems? Find the error; explain the error in this person's thought process; and correct the problem.

$x^2-4x-45=15$

$(x-9)(x+5)=15$

$(x-9)=15$ or $(x+5)=15$

$x=24$ or $x=10$

$x^3+3x^2=54x$

$\frac{x^3}{x}+\frac{3x^2}{x}=\frac{54x}{x}$

$x^2+3x=54$

$x^2+3x-54=0$

$(x-9)(x+6)=0$

$(x-9)=0$ or $(x+6)=0$

$x=9$ or $x=-6$

Homework: p. 272 – 13-15 all, 38-43 all, 47, 49-54 all, 59-64 all, 79

Section 5.4A – Square Roots

I. General Information
   A. Definition of Square Roots –
   B. Note: Square roots have more than one root.
      1. Examples – 36 has two square roots.
      2. The nonnegative root is called the principal root.
         a) $\sqrt{36}$ is asking for the principle root.
         b) $-\sqrt{36}$ is asking for the opposite of the principle root.
         c) $\pm\sqrt{36}$ is asking for the both roots.
   C. Problems – Find each root
      1. $-\sqrt{121a^6b^2}$
      2. $\pm\sqrt{169x^4}$

II. Radical Expressions
   A. Properties
      1. $\sqrt{a}\cdot\sqrt{b} =$
      2. $\dfrac{\sqrt{a}}{\sqrt{b}} =$

   B. Examples on Simplifying
      1. $-\sqrt{36a^3}$
      2. $\pm\sqrt{64x^2y^3z}$
      3. $-\sqrt{54x^4y^5z^7}$
      4. $\sqrt{60xy^3}$
      5. $2\sqrt{2}\cdot 4\sqrt{6}$
      6. $\sqrt{10x^2y}\cdot\sqrt{40xy^3}$
      7. $\sqrt{12m^2n}\cdot\sqrt{6mn}$
      8. $\sqrt{\dfrac{5}{4}}$
      9. $\dfrac{20\sqrt{8}}{2\sqrt{2}}$

III. Rationalizing the Denominator
   A. Protocol –
   B. Examples
      1. $\dfrac{1}{\sqrt{2}}$
      2. $\dfrac{5}{7\sqrt{3}}$

Homework: p. 996 – 1-32 all

Section 5.4B – Operations with Radical Expressions

I. Adding and Subtracting Radicals

A. Note:

1. To add radicals they must be like radical expressions
2. Treat _______________ like they are _______________.

B. Examples

1. $2\sqrt{3}+5+7\sqrt{3}-2$
2. $10\sqrt{2}-3\sqrt{2}+7+6\sqrt{2}$
3. $3\sqrt{27}-7\sqrt{3}-12$
4. $5\sqrt{6}-3\sqrt{24}+\sqrt{150}$

II. Multiplying Radicals

Examples

1. $\left(\sqrt{6}+\sqrt{3}\right)\left(\sqrt{3}+\sqrt{2}\right)$
2. $\left(2\sqrt{3}+4\sqrt{5}\right)\left(\sqrt{3}+6\sqrt{5}\right)$
3. $\left(12+\sqrt{3}\right)\left(12-\sqrt{3}\right)$
4. $\left(4\sqrt{5}+2\sqrt{7}\right)\left(4\sqrt{5}-2\sqrt{7}\right)$

III. Rationalizing the Denominator

Examples

1. $\dfrac{5}{2-\sqrt{6}}$
2. $\dfrac{2\sqrt{3}}{6-\sqrt{5}}$

Homework: p. 996 – 33-40 all

Section 5.4 – Complex Numbers

Goals:
1. To simplify radicals containing negative radicands.
2. To multiply pure imaginary numbers.
3. To solve quadratics equations that has pure imaginary solutions.
4. To add, subtract, and multiply complex numbers.

I. General Information

A. Rene Descartes (400 yrs ago) came up with a way to solve $x^2 = -1$.

Proposals:

1. $i = \sqrt{-1}$ where i is not a real number.
2. $i^2 = -1$

B. Pure Imaginary Numbers:

Examples

1. $\sqrt{-81} =$ and $\sqrt{-121} =$
2. Simplify: $8i \cdot 3i =$ and $\sqrt{-5} \cdot \sqrt{-20} =$
3. Simplify: $i^{12} =$ and $i^{35} =$
4. Solve: $x^2 + 81 = 0$ and $a^2 + 72 = 0$

II. Complex Numbers

A. Complex Number Form –

Real part –

Imaginary part –

Examples

1. $(8+7i)+(-12+11i)$
2. $(9-6i)-(12-2i)$
3. $(8+5i)(2-3i)$
4. $(-6+2i)(5-3i)$

III. Complex Conjugates

Examples:

1. $(2+7i)(2-7i)$
2. $(9-7i)(9+7i)$

IV. Complex Numbers in the Denominator

Examples – Simplify

1. $\dfrac{3-2i}{5-i}$
2. $\dfrac{4+5i}{3+7i}$

Homework: p. 280 – 1-17 all, 37, 42, 48, 50, 61, 66

Section 5.5 – Completing the Square

Goals:

1. To solve quadratic equations by completing the square.

Work Together

- FOIL These

1. $(x-2)^2$ 2. $(x+5)^2$ 3. $(x-7)^2$

- Making perfect squares

1. x^2-8x+ ____ $=(x-$ ___$)^2$ 2. x^2+10x+ ____ $=(x+$ ____$)^2$ 3. x^2-20x+ ____ $=(x-$ ____$)^2$

- Solve the following problems without setting equations equal to zero.

1. $x^2=36$ 2. $\left(x-3\right)^2=16$ 3. $(13+x)^2=250$

4. $x^2-4x+4=17$ 5. $x^2+6x+9=23$

- Wrap Up

Think and Discuss

- Completing the square is a method to solve quadratic equations when they do not factor.
  - Method
    1. Get constant on one side of the equation.
    2. Factor out the coefficient in front of the x^2.
    3. Make the variable side into a perfect square (reminder what ever you add to one side must be done to the other).
    4. Square root both sides and simplify.
  - Examples
    1. $x^2+10x+12=0$ 2. $x^2+11x+24=0$

    3. $6x^2-7x-5=0$ 4. $2x^2+12x+30=0$

Homework: p. 288 – 5-13 all, 27, (30-42)/3, 58, 67-69, 75-78

Section 5.6 – The Quadratic Formula and Discriminant

Goals:

2. To solve quadratic equations using the quadratic formula.
3. To use the discriminant to determine the nature of the roots of the quadratic equation.

Work Together

- Solve using the Complete the Square Method

  $ax^2 + bx + c = 0$

- Can you write a formula to solve for *x* in all quadratics?

- Wrap Up

Think and Discuss

II. The Quadratic Formula is a method to solve quadratic equations when they do not factor.

A. Formula:

If $ax^2 + bx + c = 0$, then $x = \dfrac{(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$ where $(\quad) \neq 0$

B. Examples

1. $x^2 + 10x + 12 = 0$
2. $x^2 + 11x + 24 = 0$
3. $6x^2 = 7x + 5$
4. $2x^2 + 12x + 30 = 0$

III. The Discriminant (Determines Nature of the Roots)

A. Formula: If $ax^2 + bx + c = 0$, then ____________________.

B. Translation

1. If $D > 0$, then
2. If $D = 0$, then
3. If $D < 0$, then

C. Examples – Determine the nature of the roots

1. $4x^2 = -25 + 20x$

2. $3x^2 + 2 = 5x$

Homework: p. 297 – 1-13 all, 21, 34, 43, 56 -58 all, 63

Section 5.6B – Sum and Product of Roots

Goals:

4. To find the sum and product of the roots of a quadratic equation.
5. To find all possible integral roots of a quadratic equation.
6. To find a quadratic equation to fit a given condition.

Work Together

- Solve the following quadratics

1. $x^2-2x-15=0$ 2. $x^2-\frac{5}{6}x+\frac{1}{6}=0$ 3. $x^2-14x+3=0$

- Find the sum and product of each problem's roots.

1. Sum = ________ 2. Sum =_________ 3. Sum = _________

Product = ________ Product = _________ Product = _________

- Can you make a conclusion?
(Do you see a pattern with the original quadratic and the sum and products?)

- Wrap Up

Think and Discuss

- Sum and Product Theorem:

Formula:

If the roots of $ax^2+bx+c=0$ are r_1 and r_2, then

Examples – Find the quadratic given its roots.

1. Roots are 3 and -2 2. Roots are $2/5$ and $-1/2$

*3. One root is $5+i$ *4. One root is $1+\sqrt{3}$

*Note: $a+bi$ $\left(a+b\sqrt{c}\right)$ is a root iff $a-bi$ $\left(a-b\sqrt{c}\right)$.

Homework: p. 302 – 1-10 all

Section 5.7 – Transformations with Quadratic Functions

Goals:

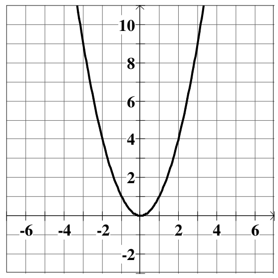
1. To graph quadratic equations of the form $y = a(x-h)^2 + k$ and identify the vertex, the axis of symmetry, and the direction of the opening.
2. To determine the equation of the parabola from given information about the graph.

I. Terms

A. Vertex –

B. Axis of Symmetry –

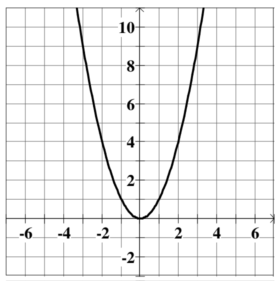
C. Parent Graph: $y = x^2$



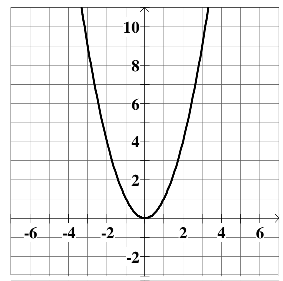
II. Dynamics of $y = a(x-h)^2 + k$

A. What does h do?

1. Graph: $y = (x-2)^2$

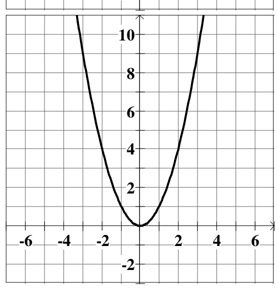


2. Graph: $y = (x+3)^2$

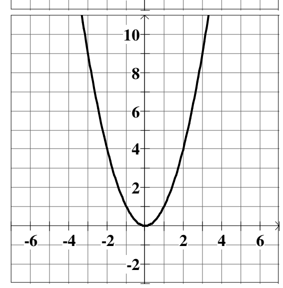


B. What does k do?

1. Graph: $y = x^2 + 4$

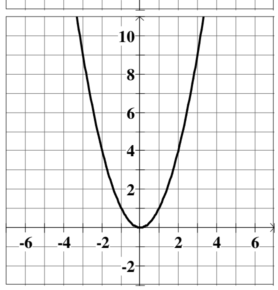


2. Graph: $y = x^2 - 3$

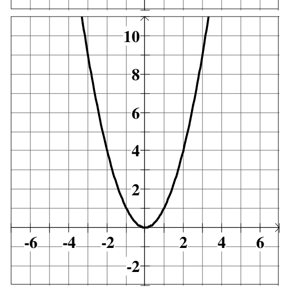


C. What does a do?

1. Graph: $y = 2x^2$



2. Graph: $y = \frac{1}{2}x^2$



Re-graph these two equations but us a negative coefficient.

D. Wrap Up $y = a(x-h)^2 + k$

1. h –
2. k –
3. a –

Vertex:

Axis of Symmetry:

E. Examples:

1. Name the vertex, axis of symmetry and direction of opening.

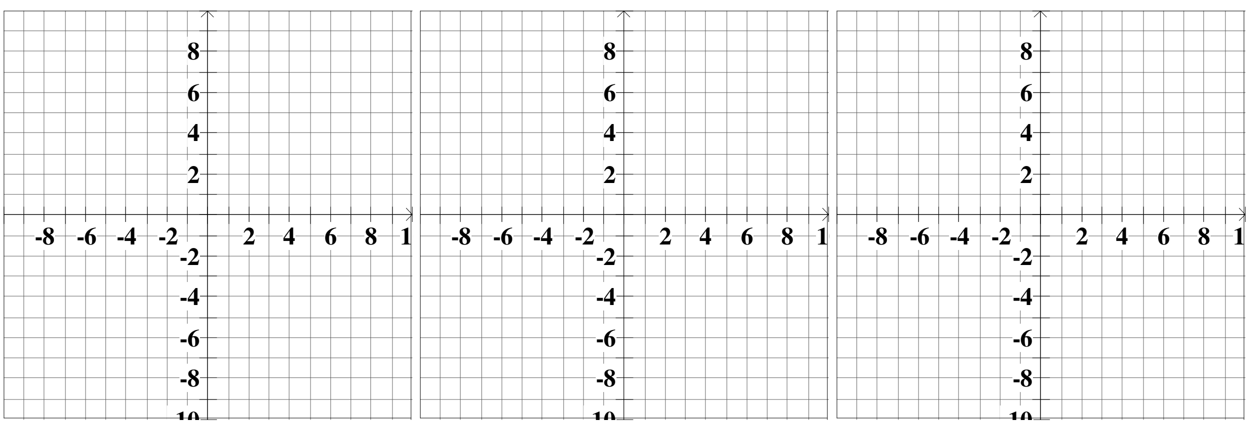
   a) $y = (x+11)^2 + 4$ b) $y = -\frac{2}{3}(x-3)^2 + 2$

2. Put the following quadratics into $y = a(x-h)^2 + k$

   a) $y = x^2 + 6x + 2$ b) $y = -4x^2 + 16x - 11$ c) $y = 0.25x^2 + 2.5x + 0.25$

3. Graph the following:

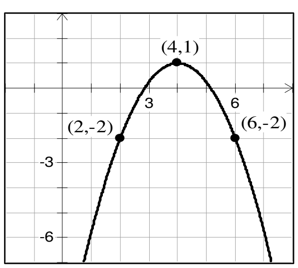
   a) $y = (x+2)^2 - 1$ b) $y = 2(x-1)^2 + 3$ c) $y = -\frac{1}{3}(x+1)^2 + 3$



III. Find the equation of the parabola

A. Examples:

1.



2. Parabola passes through the vertex (5, 4) and the point (3, –8).

Homework: p. 308 – 1-7 all, (9-18)/3, (24-33) /3, 35-39 all, 41, 44, 59, 65, 69, 72

Section 5.8 – Graphing and Solving Inequalities

Goals:

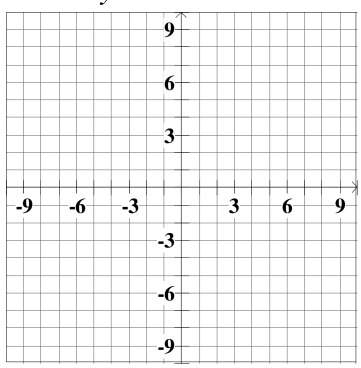
1. To graph quadratic inequalities.
2. To solve quadratic inequalities in one variable.

I. Graphing Quadratic Inequalities

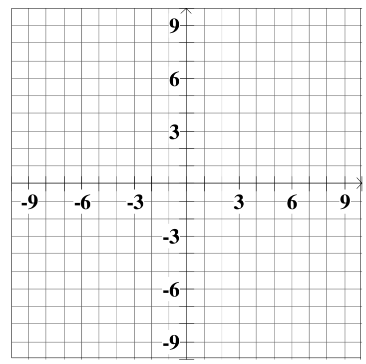
   A. Same linear functions.

   B. Examples

      1. $y < x^2 + 3x - 4$



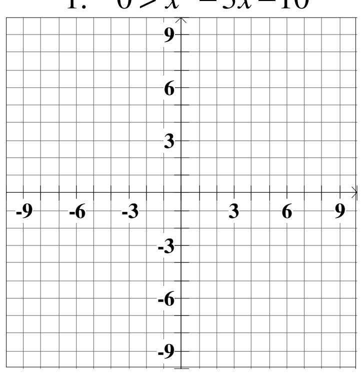
      2. $y \geq \frac{1}{2}(x-3)^2 + 1$



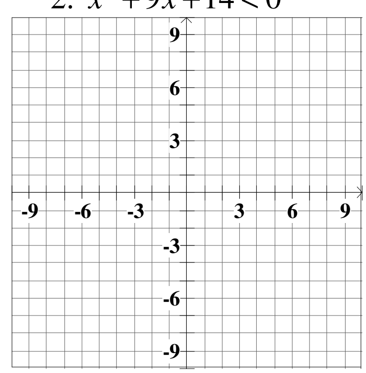
II. Solving quadratic inequalities

   A. Graphically:

      1. $0 > x^2 - 3x - 10$



      2. $x^2 + 9x + 14 < 0$



   B. Algebraically

      1. $0 > x^2 - 3x - 10$

      2. $x^2 + 9x + 14 < 0$

Homework: p. 315 – 1-12 all, 20, (33-42)/3, 47, 58, 67, 71