Section 6.1 – Operations with Polynomials

Objectives:

- Multiply, divide and simplify monomials and expressions involving powers.
- Add, subtract, and multiply polynomials.

I. Terms

A. Polynomial –

B. Terms –

C. Degree of the Polynomial –

D. Examples

E. Find the degree of the polynomials

1. $x^2 - 3x^3 + 2x$
2. $7xy^2 - 7x^2y + 8x^3y - 16xy^4$

F. Polynomial or Not?

1. $c^4 - 4\sqrt{c} + 18$
2. $-15p^5 + \frac{3}{4}p^2t^7$
3. $x^2 - \frac{3}{x} + 7$

II. Multiply and Divide Monomials

A. Properties of Exponents

1. Product of Powers:
2. Quotient of Powers:
3. Negative Exponents:
4. Power of a Power:
5. Power of a Product:
6. Power of a Quotient:
7. Zero Power:

B. Examples – Simplify the following (final answer should have no negative exponents)

1. $\left(a^{-3}\right)\left(a^2b^4\right)\left(c^{-1}\right)$

2. $\frac{n^2}{n^{10}}$

3. $\left(2x^{-3}\right)\left(-7x^5y^{-6}\right)$

4. $\frac{15c^5d^3}{-3c^2d^7}$

5. $\left(\frac{3a^3}{b^4}\right)^2$

6. $\left(\frac{a}{4}\right)^{-2}$

7. $\left(-2x^3y^2\right)^5$

8. $\frac{4xy^{-3}}{z^{-2}}$

9. $\frac{36m^3n^2}{2m^2n}$

10. $\frac{-15x^5y^{-8}(x^{-3}y^2)}{45x^{-4}y}$

11. $\left(\frac{12a^0b^5}{4b^3}\right)^3$

12. $\left(\frac{5a^{-7}b^0}{a^{-5}}\right)^{-3}$

C. Operations with Polynomials – Examples

1. $\left(4x^2-9x+3\right)+\left(-2x^2-5x-6\right)$

2. $\left(2a^3+5a-7\right)-\left(a^3-3a+2\right)$

3. $-2y^2\left(4y^2+2y-3\right)$

4. $\left(a^2+3a-4\right)\left(a+2\right)$

Homework: p. 337 – 1-15 all, 16-23 all, 29, 33, 39, 41, 49, 74, 78, 96

Section 6.3 – Polynomial Functions

Goals:

1. To evaluate polynomial functions.
2. To identify general shapes of the graphs of polynomial functions.

I. Terminology

A. Polynomials in one variable –

B. Examples: Determine if each is a polynomial of one variable.

1. $x^2 + 2xy + y^2$
2. $2a^2 - 2a + 1/4$
3. $12 - \frac{2}{n} + n^2$

C. Examples: State the degree and leading coefficient for each polynomial.

1. $7x^3 - 4x^2 + x$
2. $3x^5 + 2x^2 - 4 - 8x^6$

II. Polynomial Functions

Examples

1. Find $P(3)$ if $P(x) = 3x - 8x^2 + x^3$
2. Find $P(3) - P(-3)$ if $P(x) = 2x^2 - 4x + 6$
3. Find $P(2a)$ if $P(x) = 3x - 8x^2 + x^3$
4. Find $4P(a) + 3P(-a)$ if $P(x) = x^3 - x^2 + x$
5. Find $P(x+2)$ if $P(x) = 2x^2 + 3x + 1$

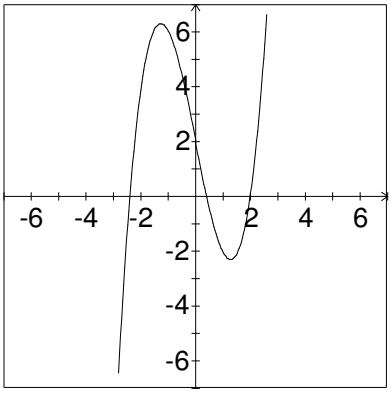
III. Polynomials and Graphs

A. Note:

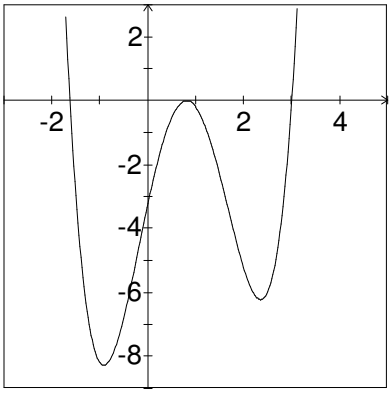
1. _____ degree polynomials – left most and right most points will be either _____ positive or negative (Superman Look)
2. Odd degree polynomials
   a) Left most points will be _____ of right most points (Swimmer Look)
   b) Will always cross the _____ at least once.
3. Real Roots
   a) Where graph crosses _____.
   b) Number of real and non-real roots is _____ to the _____ of the polynomial.
4. End Behaviors – What does the *y* value approach at the extreme _____ of the graph.
   a) Extreme right values for *x*(abbreviation: _____) what is happening to *y*?
   b) Extreme left values for *x* (abbreviation: _____) what is happening to *y*?

B. Examples: Determine if each graph is an even or odd degree polynomial, State the degree of the polynomial, state the number of real roots and non-real roots, and give the end behavior.

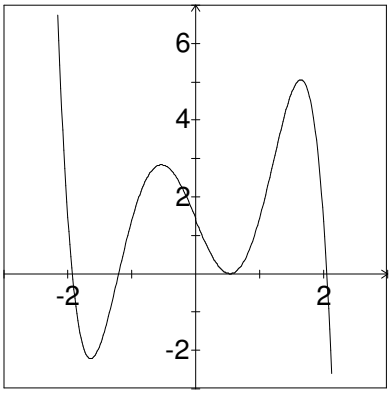
1.



2.



3.



Homework: p. 352 – 1-12 all, 27, 29, 30, 35-40 all, 47-49 all, 51, 53, 56a, 57, 63, 64

Section 6.4A– Graphing Polynomial Functions and Approximating Zeros

Goals:
1. To approximate the real zeros of polynomial functions.
2. To find relative maxima and minima of polynomial functions.
3. To graph complete graphs of polynomial functions.

I. Graphing Complete Graphs
   A. Def –
   B. Characteristics
      1. *x*- and *y*- intercepts
         a) Note: *x*-intercepts are also called _____ roots.
         b) Location Principle for real roots –
      2. Relative extrema (_____ – peeks and _____ – valleys)
   C. Examples
      1. Graph: $f(x) = -2x^3 - 5x^2 + 3x + 2$
         Give appropriate view window

      2. Graph: $f(x) = x^4 - 9x^3 + 25x^2 - 24x + 6$
         Give appropriate view window

II. Finding Characteristics
   A. Examples: Find the relative extrema, roots (*x*-intercepts), and y-intercept
      1. $f(x) = -2x^3 - 5x^2 + 3x + 2$

      2. $f(x) = x^4 - 9x^3 + 25x^2 - 24x + 6$

Homework: p. 356 – 14-21 all (state the viewing window, find the real roots, find the *y*-intercepts, and find the relative extrema), 22, 27-32 all, 34-40 all, 43, 48, 58-60 all

Section 6.4B – Modeling Real World Data with Polynomials

Goals:
1. To model data whose curve of best fit is a polynomial function.

Example:
The table at the right gives the consumption of peanuts.

Year	US Peanut Consumption (millions of pounds)
1970	1118
1980	1087
1985	1499
1990	1492
1991	1639
1992	1581
1993	1547

- Use a graphing calculator to draw a scatter plot for the data. State the viewing windows and scale factors that you used. Let 1970 = 0
- Calculate and graph curves of best fit that show how the year is related to peanut consumption. Give the equations for:
  - Linear Reg – _____
  - Quadratic Reg – _____
  - Cubic Reg – _____
  - Quartic Reg – _____
- Write the equation for the curve you think best fits the data. Explain why you think it fits the best.
  - Equation _____
  - Explain:

- Based on the cubic regression, what was the consumption in 1994? _____.
  Can I use this equation to predict consumption in 2010? Explain.

  Can I use this equation to estimate consumption in 1960? Explain.

  When can I use this equation? Explain.

- Based on the quartic regression, when will consumption be 1500 million pounds? _____

Homework: p. 366 – 1-13 all

Section 6.5A –Solve Polynomial Equations

Goals:
1. Factor Polynomials
2. Solve polynomials by factoring.

Examples

1. Solve: $x^4 + 3x^3 - 18x^2 = 0$.

2. Solve: $x^3 + 5x^2 - 2x - 10 = 0$.

3. Solve: $t^3 - 216 = 0$.

Difference/Sum of Two Cubes
Formulas: 1. $a^3 - b^3 =$ 2. $a^3 + b^3 =$

4. Solve: $8x^3 + 27 = 0$.

5. Solve: $x^4 - 29x^2 + 100 = 0$.

6. Solve: $x - 13\sqrt{x} + 36 = 0$.

7. Solve: $2x^6 - 3x^3 + 9 = 0$.

Homework: p. 372 – 30-35all, 42-47all, 59-69 odds, 80,

Section 6.5B – Solving polynomial Inequalities (Graphically)

Goals:

- Use a graphing calculator to find solutions for polynomial inequalities.

Examples:

1. $x^4 + 2x^3 \leq 7$

2. $9x^4 - 27x^2 + 20 > 0$

3. $\frac{1}{2}x^3 + x^2 - 5x \leq -13$

Homework: p. 376 – 1-9 all

Section 6.2 – Dividing Polynomials

Goals:
1. To divide polynomials using long division.
2. To divide polynomials using synthetic division.

I. Dividing Polynomials by monomials

Examples

1. $\dfrac{3a^2b+6a^3b^2+18ab}{3ab}$

2. $\dfrac{12x^2y+3x}{4x}$

II. Dividing Polynomials by Polynomials

A. Long Division

Examples

1. $x+3\overline{)x^2-5x-24}$

Note:
When dividing make sure the polynomial is in descending order.

2. $\left(x^2-11x+28\right)\div\left(x-4\right)$

3. $\left(x^3-27\right)\left(x-3\right)^{-1}$

4. $\dfrac{x^6+5x^3+x-1}{x^2+3}$

III. Synthetic Division

A. Def – representation of division where you are only using the coefficients.

B. Note can only be used when dividing by $x \pm a$

C. Example

1. $\left(3x^3 - x^2 + 2x - 4\right) \div (x + 3)$

$$x+3\overline{)3x^3 - x^2 + 2x - 4}$$

-3	3	-1	2	-4
	↓			

2. $\left(x^4 - 2x^3 + x^2 - 3x + 2\right) \div (x - 2)$

3. $\left(x^4 - 2x^3 + x - 1\right) \div (x + 1)$

4. $\left(6x^2 - 2x + 4\right) \div (2x - 3)$

Homework: p. 345 – 1-8 all, 21-31 odds, 55, 57, 58, 63-65 all

Section 6.6 – The Remainder and Factor Theorems

Goals:

1. To find factors of polynomials using the factor theorem and synthetic division.

I. Work Together

A. Find the remainder to each problem.

1. $\dfrac{x^4-6x^3+8x+5x+13}{x-4}$

2. $\dfrac{3x^5-3x^3+57}{x+2}$

B. Find the value for each function

1. Find $f(4)$ if $f(x)=x^4-6x^3+8x+5x+13$.

2. Find $f(-2)$ if $f(x)=3x^5-3x^3+57$.

C. Can you make a conclusion?

D. Wrap Up

II. Think and Discuss

A. Remainder Theorem: If $f(x)\div(x-a)$, then $f(a)$ equals the remainder of $f(x)\div(x-a)$.

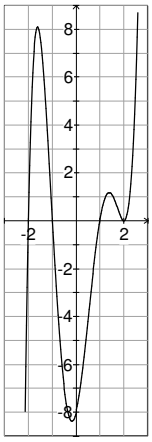
B. Factor Theorem

1. Theorem: $(x-a)$ is a factor of $f(x)$ iff $f(a)=0$
2. Examples:

a) Is $x-2$ a factor of x^4-x^3+2x-2?

b) Is $x-2$ a factor of $x^3+7x^2+2x-40$?
If so find all the factors.

c) Find the polynomial
(in factored form) for the given graph.



Homework: p. 380 – 1-7 all, 17-23 odds, 29, 30, 33, 35, 36, 38, 39

Section 6.7 and 6.8 – Finding all the Roots (Real and Non-real)

Goal:

1. To find all the real and non-real roots of a polynomial.
2. To find all the factors of a polynomial.
3. To write the polynomial of least degree given zeros.

Examples:

1. Find all the factors of $f(x) = x^3 + x^2 - 4x - 4$ given $(x + 2)$ is a factor.

2. Find all the roots of $f(x) = x^3 - 5x^2 - 7x + 51$ given that $4 - i$ is a root.

3. Write the polynomial function of least degree with integral coefficients whose zeros include the following.
   - 1 and $1 + i$

   - -2 and $2 + \sqrt{3}$

Homework: worksheet, p. 389 – 50-52 all, 54, 55, 61