Work Together

- How many pounds of peanuts do you think and average person consumed last year?
  - Us the table at the right. What was the average peanut consumption per person in the US in 1993?

Year	US Peanut Consumption (millions of pounds)	US Population (millions)
1970	1118	203.3
1980	1087	226.5
1985	1499	237.9
1990	1492	248.7
1991	1639	252.1
1992	1581	255.0
1993	1547	257.8

- You can use the data in the table to model the average US consumption per person between 1970 and 1993.
  - Let x = 0 represent 1970. Input the data into three lists on your calculator.
  - Find a cubic function $f(x)$ to model the annual consumption of peanuts.
  - Find a quadratic function $g(x)$ to represent the US population since 1970.
  - Write a function $h(x)$ to model the average peanut consumption in the US since 1970.
  - Graph the function on your calculator you wrote to estimate the number of peanuts that were consumed by the average person last year.

- Wrap Up
  - A rational function occurs when one polynomial is divided by another polynomial.
  - Example: $f(x) = \dfrac{3x^2 - 1}{(x-3)(x+4)}$

Section 9.1 – Multiplying and Dividing Rational Expressions

Goals:

1. To simplify rational expressions.
2. To determine when a rational function is undefined.
3. To simplify complex fractions.

Examples

1. Simplify and determine when the rational expression is undefined:

   a. $\dfrac{3y(y+7)}{(y+7)(y^2-9)}$ b. $\dfrac{x(x+5)}{(x+5)(x^2-16)}$

2. For what values of p is $\dfrac{p^2+2p-3}{p^2-2p-15}$ undefined?

3. Simplify: $\dfrac{a^4b-2a^4}{2a^3-a^3b}$

4. Multiply or Divide and Simplify:

   a. $\dfrac{8x}{21y^3}\cdot\dfrac{7y^2}{16x^3}$ b. $\dfrac{10mk^2}{3c^2d}\div\dfrac{5m^5}{6c^2d^2}$

   c. $\dfrac{3x}{15y}\cdot\dfrac{5y^2}{2x^3}$ d. $\dfrac{3x^2y}{20ab}\div\dfrac{6xy}{5a^2b^3}$

5. Simplify:

a. $\frac{k-3}{k+1}\cdot\frac{1-k^2}{k^2-4k+3}$

b. $\frac{2d+6}{d^2+d-2}\div\frac{d+3}{d^2+3d+2}$

c. $\frac{x-3}{x+2}\cdot\frac{x^2+5x+6}{x^2-9}$

d. $\frac{3d+9}{d^2+4d+3}\div\frac{d+2}{d^2+5d+4}$

6. Simplify the complex fraction:

a. $\dfrac{\frac{x^2}{9x^2-4y^2}}{\frac{x^3}{2y-3x}}$

b. $\dfrac{\frac{a^2}{a^2-9b^2}}{\frac{a^4}{a+3b}}$

Simplifying Error: $\boxed{\frac{2x+\cancel{5}^{1}}{\cancel{15}_{3}}=\frac{2x+1}{3}}$ Very Bad News!!!!!!

Note: You cannot simplify through addition or subtraction.

Homework: p. 557 – 1-12 all, 19-23 all, 38, 46-48 all, 60, 71-79 odds

Section 9.2 – Adding and Subtracting Rational Expressions

Goals:

1. To find the least common denominator.
2. To add and subtract rational expressions.

A. Find the Least Common Multiple (or LCD)

Examples

1. $15a^2bc^3$, $16b^5c^2$ and $20a^3c^6$

2. $x^3 - x^2 - 2x$ and $x^2 - 4x + 4$

3. $6x^2zy^3$, $9x^3y^2z^2$, and $4x^2z$

4. $x^3 + 2x^2 - 3x$ and $x^2 + 6x + 9$

B. Adding and Subtracting Rational Expressions

Examples

1. $\dfrac{5a^2}{6b} + \dfrac{9}{14a^2b^2}$

2. $\dfrac{3x^2}{2y} + \dfrac{5}{12xy^2}$

3. $\dfrac{x+10}{3x-15} - \dfrac{3x+15}{6x-30}$

4. $\dfrac{x+5}{2x-4} - \dfrac{3x+8}{4x-8}$

C. Simplify Complex Fractions

1. $\dfrac{\frac{2}{x} - 1}{\frac{1}{y} - \frac{3}{x}}$

2. $\dfrac{\frac{3}{a} - \frac{4}{b}}{\frac{2}{b} - \frac{1}{ab}}$

Homework p. 565 – 1-17 all, 29, 31, 37, 51, 59, 71-74 all

Section 9.3 – Graphing Reciprocal Functions

Goals:

1. Determine properties of reciprocal functions.
2. Graph transformations of reciprocal functions.

I. Reciprocal Functions

A. Key Concept:

Key Concept — **Parent Function of Reciprocal Functions** — For Your FOLDABLE

Parent function: $f(x) = \frac{1}{x}$

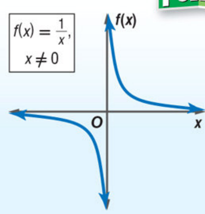
Type of graph: **hyperbola**

Domain and range: all nonzero real numbers

Axes of symmetry: $x = 0$ and $f(x) = 0$

Intercepts: none

Not defined: $x = 0$ and $f(x) = 0$



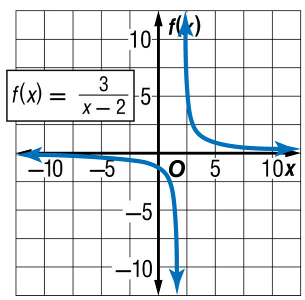
B. Asymptotes: The ______________ lines a rational functions approach.

1. Vertical Asymptote:
2. Horizontal Asymptote:

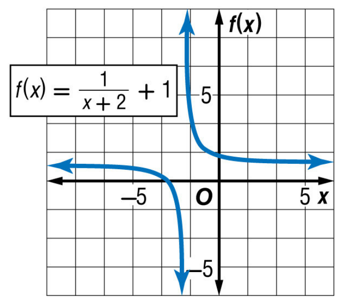
C. Examples:

Identify the asymptotes, domain, and range of the function.

1.



2.



II. Transformations of Reciprocal Functions

B. Key Concept:

Key Concept — **Transformations of Reciprocal Functions** — For Your FOLDABLE

$$f(x) = \frac{a}{x - h} + k$$

***h* – Horizontal Translation**	***k* – Vertical Translation**
$\lvert h \rvert$ units right if h is positive $\lvert h \rvert$ units left if h is negative The *vertical* asymptote is at $x = h$.	$\lvert k \rvert$ units up if k is positive $\lvert k \rvert$ units down if k is negative The *horizontal* asymptote is at $f(x) = k$.

***a* – Orientation and Shape**

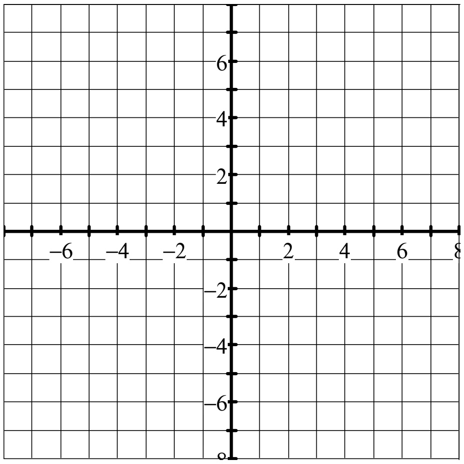
If $a < 0$, the graph is reflected across the x-axis.

If $\lvert a \rvert > 1$, the graph is stretched vertically. If $0 < \lvert a \rvert < 1$, the graph is compressed vertically.

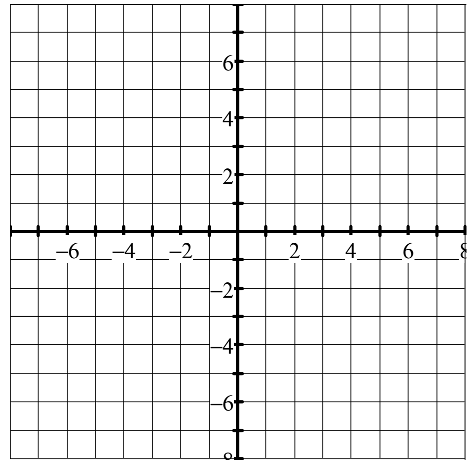
C. Examples

Graph the following functions and state their domain and range.

1. $f(x)=\dfrac{-1}{x+1}+3$

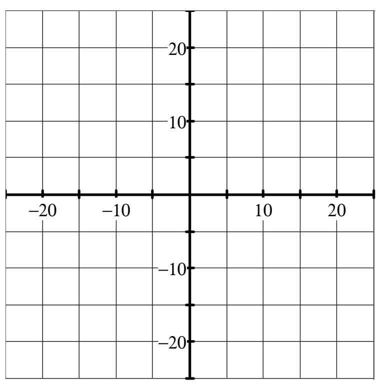


2. $f(x)=\dfrac{2}{x-3}+3$



3. A commuter train has a nonstop service from one city to another, a distance of about 25 miles.

   a) Write an equation to represent the travel time between these two cities as a function of rail speed. Then graph the equation.



   b) Explain any limitations to the range and domain in this situation.

Homework: p. 572 – 1-6 all, 10, 15, 24, 49-59 odds

Section 9.4 – Graphing Rational Functions

Goal:

1. To graph rational functions with vertical and horizontal asymptotes.
2. To graph rational functions with slant asymptotes and point discontinuity.

I. Horizontal and Vertical Asymptotes

A. Reminder of Rational Functions

> If $f(x) = \frac{a(x)}{b(x)}$, $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1, and $b(x) \neq 0$, then:

B. Vertical Asymptotes

> $f(x)$ has a **vertical asymptote** whenever $b(x) = 0$.

Note – One exception (point discontinuity) holes in the graph.

C. The horizontal asymptotes

> $f(x)$ has at most one **horizontal asymptote**.
>
> - If the degree of $a(x)$ is greater than the degree of $b(x)$, there is no horizontal asymptote. Produces a slant asymptote.
> - If the degree of $a(x)$ is less than the degree of $b(x)$, the horizontal asymptote is the line $y = 0$.
> - If the degree of $a(x)$ equals the degree of $b(x)$, the horizontal asymptote is the line $y = \frac{\text{leading coefficient of } a(x)}{\text{leading coefficient of } b(x)}$.

D. Examples:

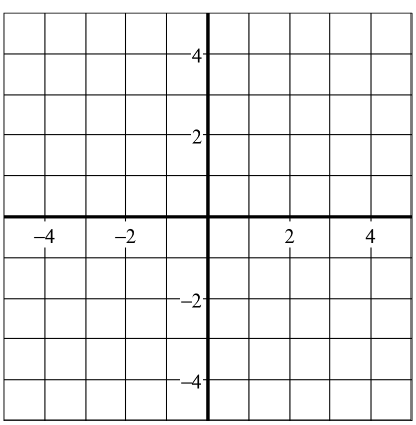
1. Determine the equations of any vertical and horizontal asymptotes and the values of x for any holes in the graph of:

a) $f(x) = \frac{x^2 - 4}{x^2 + 5x + 6}$

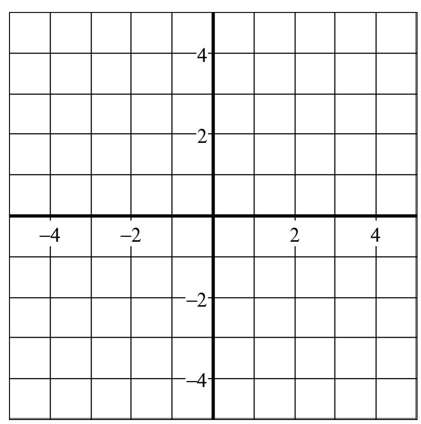
b) $f(x) = \frac{3x^2 - 27}{x^2 + 8x + 15}$

2. Graph and name the asymptotes:

a) $f(x) = \frac{x}{x+1}$

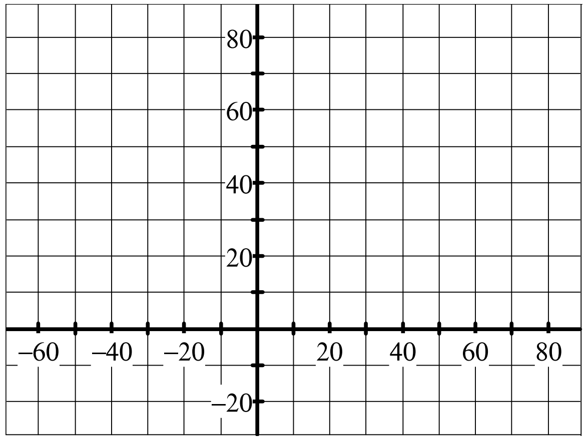


b) $f(x) = \frac{x}{x+3}$



3. A boat traveled upstream at r1 miles per hour. During the return trip to its original starting point, the boat traveled at r2 miles per hour. The average speed for the entire trip R is given by the formula $R = \dfrac{2r_1r_2}{r_1 + r_2}$.

   a) Draw the graph if $r_2 = 15$ miles per hour.



   b) What is the R-intercept of the graph?

   c) What domain and range values are meaningful in the context of the problem?

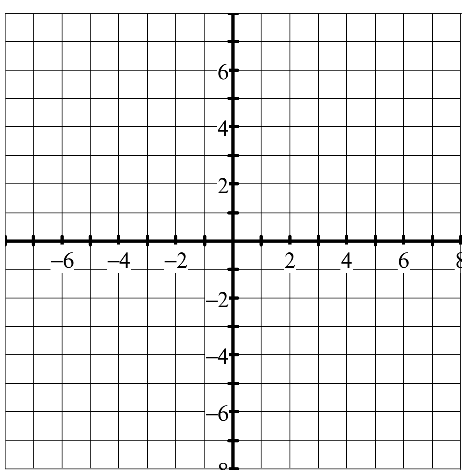
II. Slant Asymptotes

A. Key Concept

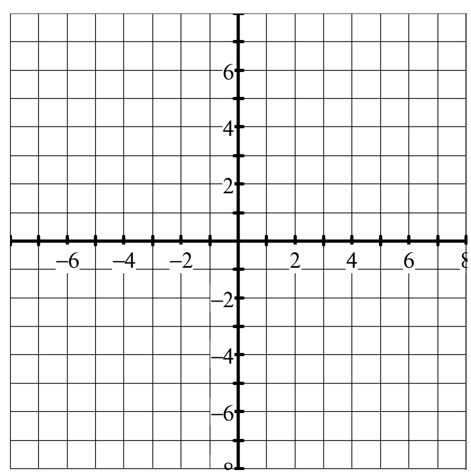
If $f(x) = \dfrac{a(x)}{b(x)}$, $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1 and $b(x) \neq 0$, then $f(x)$ has an oblique asymptote if the degree of $a(x)$ minus the degree of $b(x)$ equals 1. The equation of the asymptote is $\dfrac{a(x)}{b(x)}$ with no remainder.

B. Example:

1. $f(x) = \dfrac{x^2}{x+1}$



2. $f(x) = \dfrac{x^2 - 3x - 10}{x - 4}$



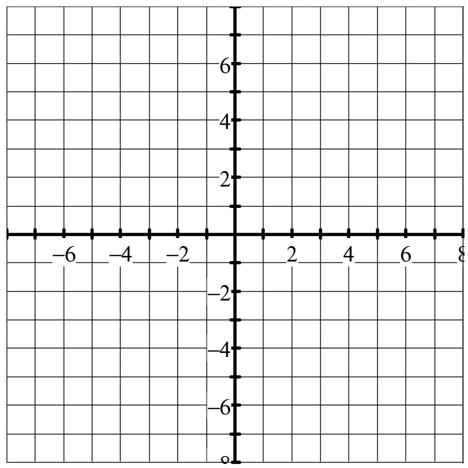
III. Point Discontinuity

A. Key Point

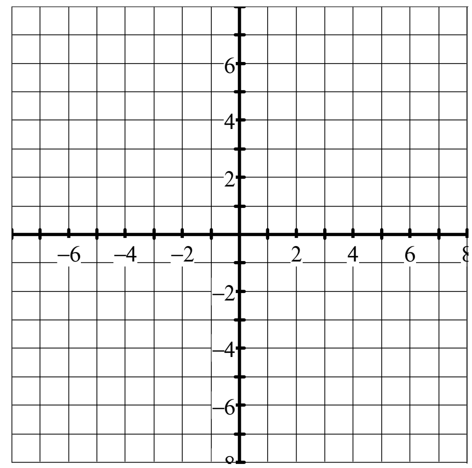
If $f(x) = \frac{a(x)}{b(x)}$, $b(x) \neq 0$, and $x - c$ is a factor of both $a(x)$ and $b(x)$, then there is a point discontinuity at $x = c$.

B. Example:

1. $f(x) = \dfrac{x^2 - 4}{x - 2}$



2. $f(x) = \dfrac{x^2 - 16}{x + 4}$



Homework p. 581 – 3, 13, 14, 16, 23, 38, 39, 41, 51-61 odds

Goals:
1. To solve problems involving direct and joint variations.
2. To solve inverse and combined variation problems

A. Direct Variation
   1. As x ___________ (_________), y ___________ (__________).
   2. Wording –
   3. General Equation: $y = kx$ where k is the constant of variation.
   4. This is similar to the ___________ equation $y = mx$ where m is the slope.
   5. Example:
      a) If y varies directly as x and $y = -15$ when $x = 5$, find y when $x = 3$.

Method to solve variation problems:
1.
2.
3.
4.

b) If y varies directly as x and $y = 12$ when $x = -3$, find y when $x = 7$.

B. Joint Variation
   1. Wording –
   2. General Equation: $y = kxz$ where k is the constant of variation.
   3. Example:
      a) The volume of a cone varies jointly as the square of the radius of the base and the height. Find the equation of joint variation if $V = 285$, $r = 4$, and $h = 17$.

      b) Suppose y varies jointly as x and z. Find y when $x = 3$ and $z = 2$, if $y = 11$ when $x = 5$ and $z = 22$.

C. Inverse Variation

1. As x __________ (__________), y ___________ (___________)
2. Wording –
3. General Equation: $y = \frac{k}{x}$ where k is the constant of variation.
4. Examples
   a) If y varies inversely as x and $y = 2$ when $x = 6$, find y when $x = -7$.

   b) If temperature is constant, gas volume varies inversely with its pressure. If an air-filled balloon has a volume of $2.6\ dm^3$ when pressure is 120 kpascals, what is the pressure if the volume is $3.2\ dm^3$?

D. Combined Variations

1. The force is directly related to the product of the masses of two objects and inversely related to the square of the distance between them.

2. Suppose f varies directly as g, and f varies inversely as h. Find g when $f = 6$ and $h = -16$, if $g = 10$ when $h = 4$ and $f = -6$.

Homework p. 590 – 1-6 all, 11, 16, 21, 22, 25-27 all, 44, 57-59 all, 61-65 odds

Section 9.6 – Solving Rational Equations and Inequalities

Goals:

1. To solve rational equations and inequalities

Examples:

1. $\frac{5}{24}+\frac{2}{3-x}=\frac{1}{4}$

2. $\frac{5}{2}+\frac{3}{x-1}=\frac{1}{2}$

3. $\frac{p^2-p+1}{p+1}=\frac{p^2-7}{(p-1)(p+1)}+p$

4. $\frac{1}{x-2}=\frac{2}{x+4}-\frac{2x}{x^2+2x-8}$

5. Aaron adds an 80% brine (salt and water) solution to 16 ounces of solution that is 10% brine. How much of the solution should be added to create a solution that is 50% brine?

6. Janna adds a 65% base solution to 13 ounces of solution that is 20% base. How much of the solution should be added to create a solution that is 40% base?

7. Tim and Ashley mow lawns together. Tim working alone could complete the job in 4.5 hours, and Ashley could complete it alone in 3.7 hours. How long does it take to complete the job when they work together?

Note:

Work = __________ x _________

Rate = How much of the ________ can be ___________ in a given _________ of ___________.

$W_1+W_2+W_3\cdots=W_{Total}$

8. Libby and Nate clean together. Nate working alone could complete the job in 3 hours, and Libby could complete it alone in 5 hours. How long does it take to complete the job when they work together?

9. A car travels 300 km in the same time that a freight train travels 200 km. The speed of the car is 20 km/h faster than the train. Find the speed of the train.

Note:
Distance = _________ x _________

Key Concept — **Solving Rational Inequalities** — For Your FOLDABLE

Step 1 State the excluded values. These are the values for which the denominator is 0.

Step 2 Solve the related equation.

Step 3 Use the values determined from the previous steps to divide a number line into intervals.

Step 4 Test a value in each interval to determine which intervals contain values that satisfy the inequality.

10. Solve: $\frac{1}{3k} + \frac{2}{9k} < \frac{2}{3}$

11. Solve: $\frac{1}{x} + \frac{3}{5x} < \frac{2}{5}$

Homework p. 600 – 1-15 all, 22-24 all, 47, 49, 50, 51