Section 6.1 – Angles and Radian Measure

- Review
  - If you measured the distance around a circle in terms of its radius, what is the unit of measure?
  - In relationship to a circle, if I go half way around the edge of a circle how many _____ is it?
  - With what you just learned, how many degrees of rotation are there for $\frac{\pi}{4}$ _____ of measure?
  - How many _____ of measure are there for a 150° rotation?
  - What are the major equivalents:

Degrees	Radians	Sine	Cosine	Tangent
0°				
30°				
45°				
60°				
90°				

  - Evaluate $\tan\left(\frac{25\pi}{4}\right)$, $\sin\left(\frac{7\pi}{4}\right)$, and $\sec\left(\frac{-13\pi}{6}\right)$

- Other Items
  - Central Angle – An angle whose vertex lies at the center of the circle.
  - Circular Arc
    - Defined by the central angle it intercepts.
    - Radian measure is used to find the length of the arc.
    - Examples:
      1. Given a central angle of 125°, find the length of its intercepted arc in a circle of radius 7 cm.

      2. Given a central angle of $\frac{2\pi}{3}$ rad, find the length of its intercepted arc in a circle of radius 5 cm.

- Sector – a region bounded by a central angle and the intercepted arc.
- Examples:

1. Find the area of a sector if the central angle measures $\frac{3\pi}{7}$ rad and the radius of 11 cm.

2. In the universe, the most common elements are hydrogen (73.9%), helium (24.0%), oxygen (1.07%), and all others combined (1.03%). On a circle graph with a radius of 1 inch, what area of the circle represents hydrogen?

Homework: p. 348 –(18-57)/3, 60, 62-66 all, 68, 70-72 all

Section 6.2 – Linear and Angular Velocity

Work Together

- Suppose I had a 26-inch wheels on my bicycle and they rotated at 200 rpm, how fast am I traveling linearly in inches/second?

- Can you give me a formula that can be used to find the linear velocity so we can save time?

  Linear Velocity =

- Key Terms
  - Angular Displacement – As any _____ object rotates about its _____, an object at the _____ moves through an _____ relative to its starting position known as _____ displacement.
  - Angular Velocity – the _____ in the central _____ with _____ to _____ as an object moves along a circular path.
    Forms of measurement
    -
    -
    -

- Key Conversions
  -
  -
  -

- Examples:

1. Determine the angular displacement in radians of 8.7 revolutions.

2. Determine the angular velocity if 5.8 revolutions are completed in 9 seconds.

3. A pulley of radius 20-cm turns at 6 revolutions per second. What is the linear velocity of the belt driving the pulley?

4. A truck driver drives 55-mph. His truck's tires have a radius of 24 inches. What is the angular velocity of the wheels in rps?

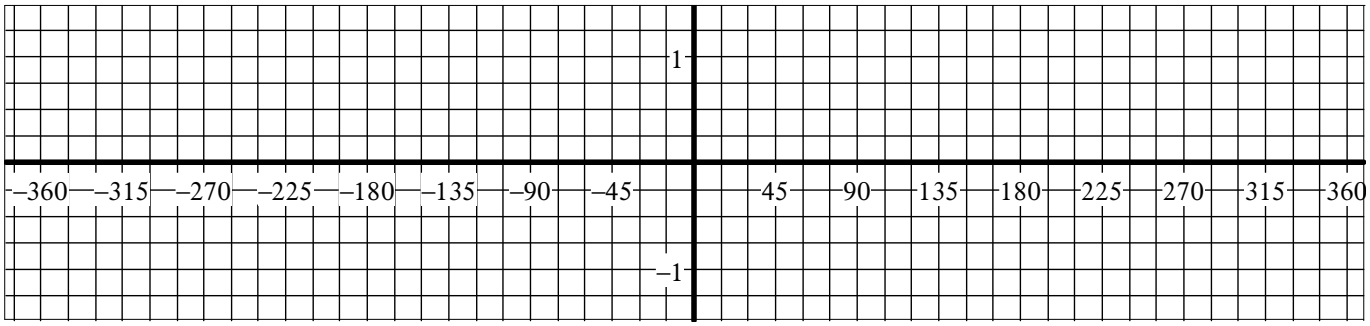
Homework: p 355 – 3-5all, (15-36)/3, 37, 40, 41, 43, 44-51 all

Section 6.3 – Graphs of Trig Functions

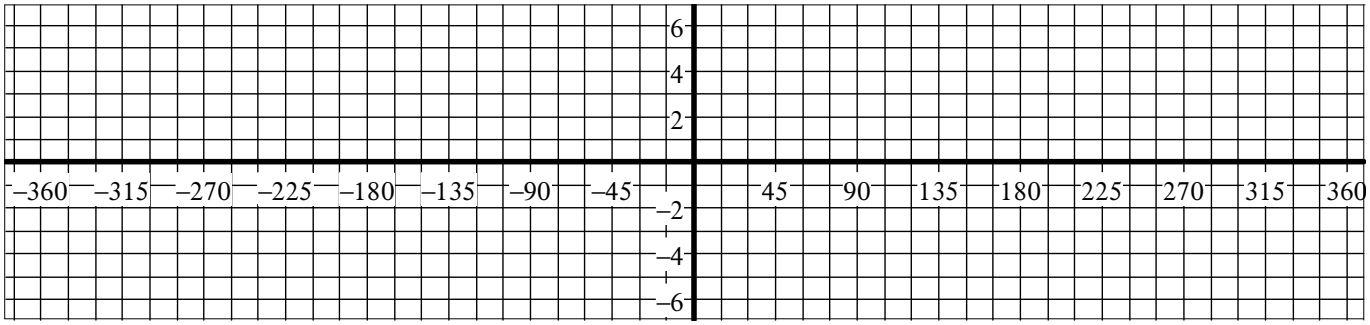
Objectives:
1. To graph basic trig functions using t-bar method.

A. Sine and Cosecant

1. $y = \sin\theta$

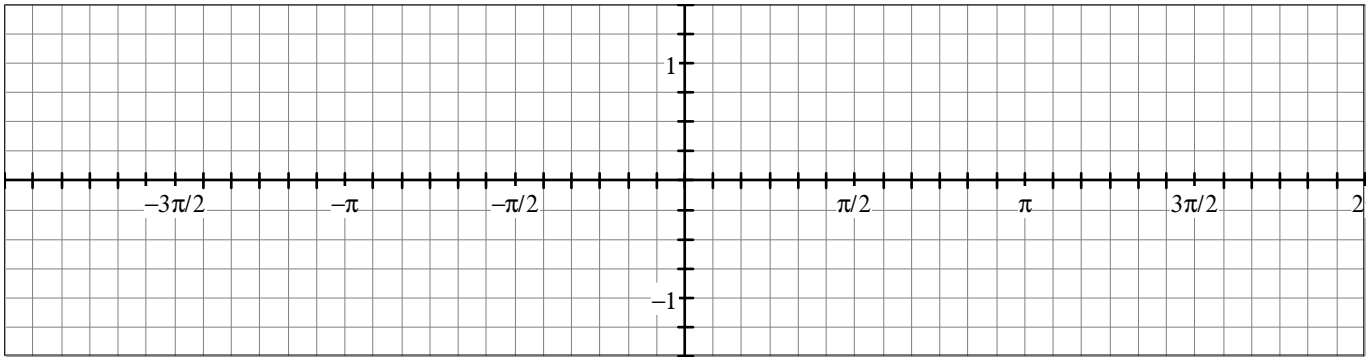


2. $y = \csc\theta$

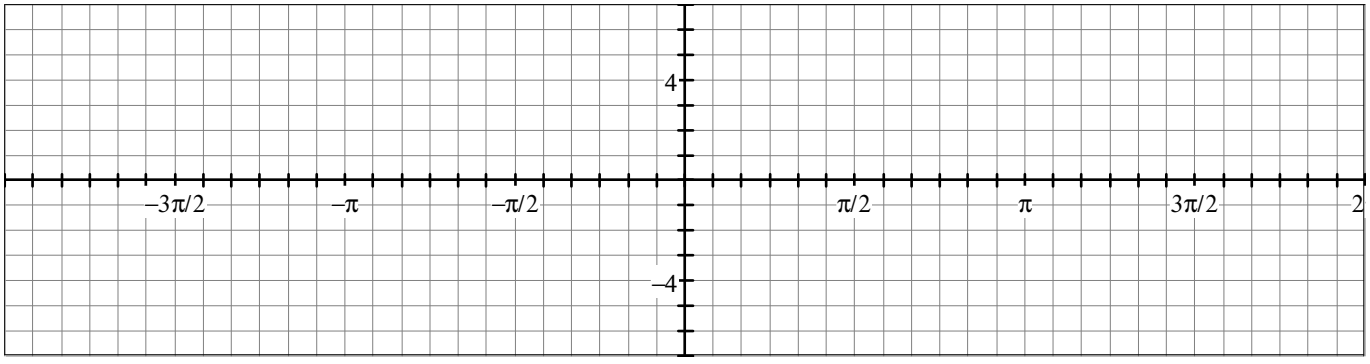


B. Cosine and Secant

1. $y = \cos x$

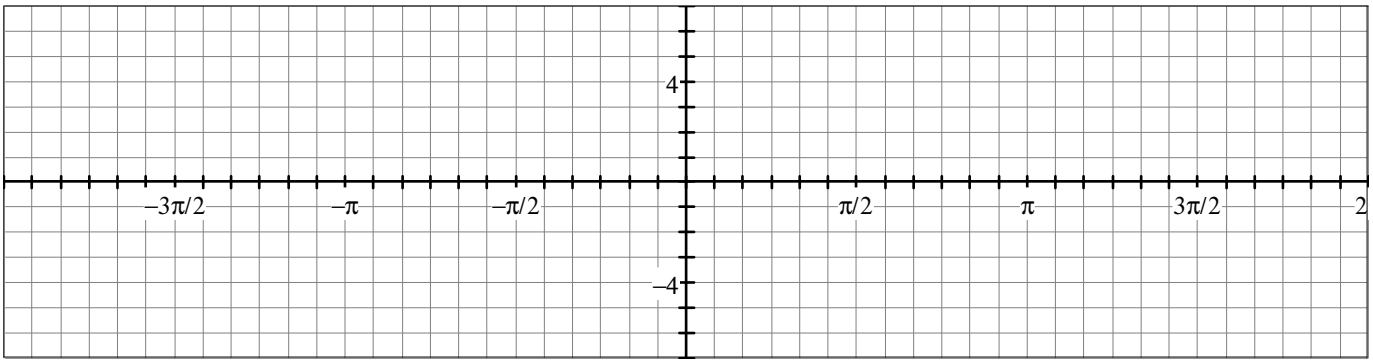


2. $y = \sec\theta$

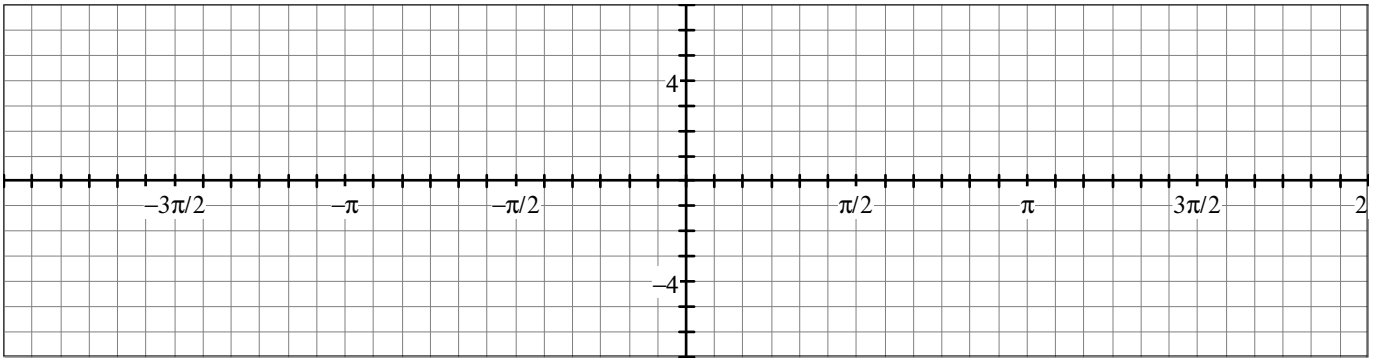


C. Tangent and Cotangent

1. $y = \tan x$



D. $y = \cot x$



Homework: p. 363 – 5-18 all, 57-60 all, 65, 67, 68

Section 6.4 – Amplitude and Period

Objectives:
1. To find the amplitude and period for a trigonometric function.
2. To write equations of trigonometric functions given amplitude and period.

I. Graphing Equations

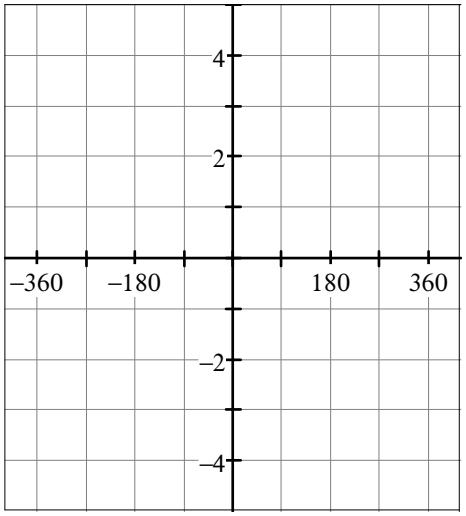
A. Forms:

1.

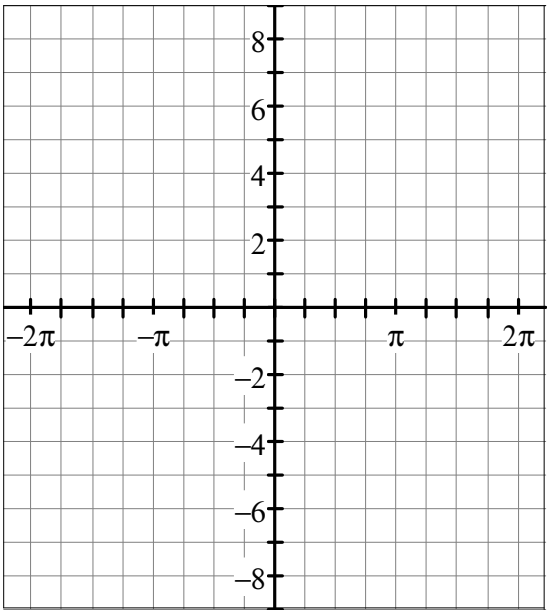
2.

B. What does $|A|$ affect?

1. Graph:
   a) $y = 3\sin\theta$
   b) $y = -5\cos\theta$
2. Amplitude $(|A|)$ – Distance from _____.
3. Example:
   a) Graph manually: $y = 2.5\sin\theta$



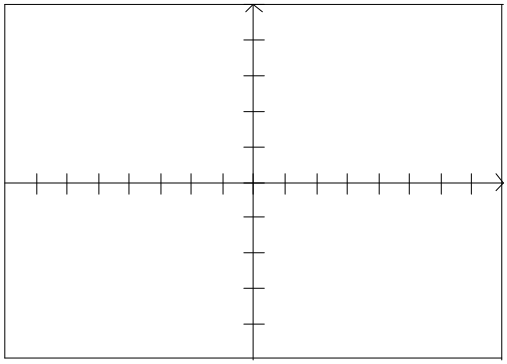
   b) What is the amplitude of $y = -7\cos x$?
4. How does $-A$ affect the graph?
   a) Graph: $y = -3\cos x$ and $y = 3\cos x$
   b) Graph: $y = -3\sin x$ and $y = 3\sin x$
   c) Graph manually: $y = -7\cos x$



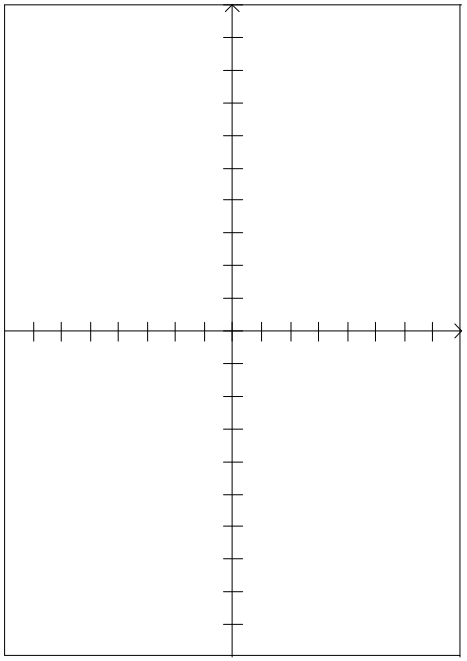
C. What does $|B|$ affect?

1. Graph:
   a) $y = \sin 3x$
   b) $y = \cos(-2x)$
2. $(|B|)$ -- _____ of _____ in $360°$ or 2π.
3. Period – _____ to complete _____ cycle $\left(period = \text{——} \text{ or } \text{——} \right)$

   (Note: tangent is in $180°$ or π.)
4. Examples:
   a) Graph: $y = \cos\frac{1}{2}x$ (Note: When graphing, break your cycle into 4 equal parts)

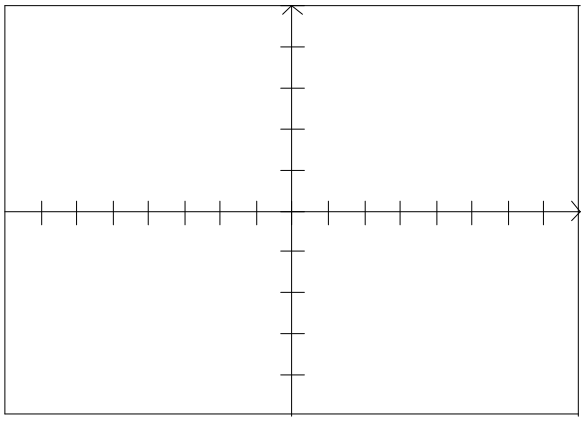


   b) Graph: $y = \tan 2\theta$



5. How does $-B$ affect the graph?
   a) Graph: $y = \cos(-3x)$ and $y = \cos 3x$
   b) Graph: $y = \sin(-3x)$ and $y = \sin 3x$

6. Graph manually: $y = \cos\left(-\frac{1}{4}\theta\right)$



D. Doing Both Amplitude and Period

1. Graph manually: $y = 5\cos 3x$

2. Graph manually: $y = -2\sin\frac{1}{2}x$

3. Graph manually: $y = 2\sin\left(-\frac{1}{3}\theta\right)$

II. Writing Equations:

Examples:

1. Amplitude = 2 and Period = 90°

2. The motion of a prong of a tuning fork can be described by a modified trig function. Represent displacement of a prong from the rest positions to the right as positive and displacement to the left as negative. At time zero, a prong reaches its maximum displacement, 0.03-cm to the right. The fork has a frequency of 200 cycles per second.

   a) Write a function that represents the position of the prong, *y*, in reference to its rest position, in terms of the time, *t*, measured from $t = 0$ seconds.

   b) Find *y*, when $t = 0.02$ seconds

Homework: p.372 – 2-4 all, (6-54)/3, 56, 58, 59, 61, 63-68 all, 73

Section 6.5 – Phase Shift and Translations

Objectives:
1. To find the phase shift and translations for a trigonometric function.
2. To graph trig functions with phase shifts and translations
3. To write equations of trigonometric functions given phase shifts and translations.

I. Graphing Equations

A. Forms:

1.
2.

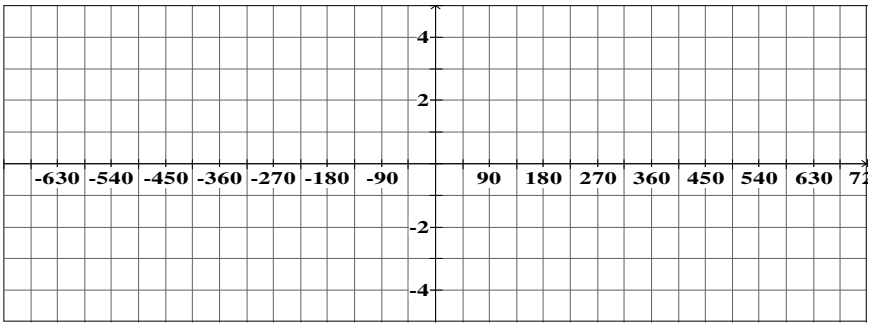
B. What does *D* affect?

1. Graph:
   a) $y = \sin(\theta - 60°)$
   b) $y = \cos(x + \pi)$
2. Phase Shift (*D*) – _____ the vertical_____ axis _____ or _____.
3. How does -*D* affect the graph?

   Graph: $y = \cos\left(x - \frac{\pi}{4}\right)$ and $y = \cos\left(x + \frac{\pi}{4}\right)$

   a)
   b)

4. Examples:
   a) Graph manually: $y = \sin(\theta - 45°)$



   b) What is the phase shift of $y = \cos(\theta + 180°)$?

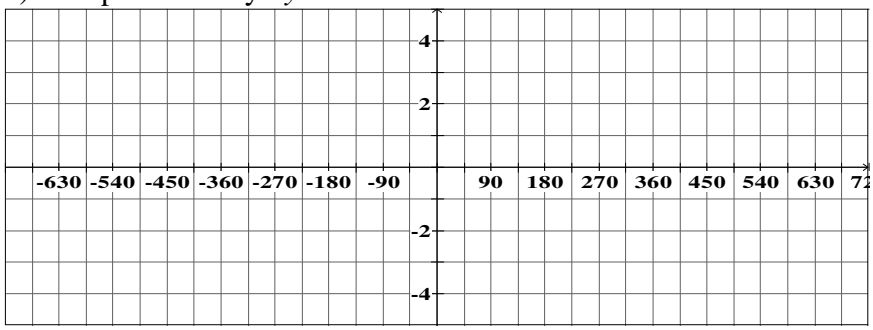
C. What does *C* affect?

1. Graph:
   a) $y = 3 + \sin x$
   b) $y = -2 + \cos x$
2. Translation (*C*) – translating _____ _____ or _____.
3. How does -*C* affect the graph?

   Graph: $y = -3 + \cos x$ and $y = 3 + \cos x$

   a)
   b)

4. Examples:
   a) Graph manually: $y = -2 + \cos x$



   b) What is the translation of $y = 5 - \sin\theta$?

D. Doing Both Phase Shift and Translation
   1. Graph manually: $y = 5 + \cos(x - \pi)$

   2. Graph manually: $y = -2 + \sin(\theta + 15°)$

E. Basic Type of Transformations Review
   1. Original Graph: $y = C + A\sin B(x - D)$

      Rewrite using *r*

   2. Horizontal shift *r* units to the right: _____
   3. Horizontal shift *r* units to the left: _____
   4. Vertical shift *r* units to the down: _____
   5. Vertical shift *r* units to the up: _____
   6. Reflection (about the x-axis): _____
   7. Reflection (about the y-axis): _____

F. Doing it all
   1. Graph manually: $y = 5 - 3\cos\frac{1}{2}(x - \pi)$

2. Graph manually: $y = -2 + 6\sin\left(-\frac{1}{4}\theta + 15°\right)$

3. Graph manually: $y = -2 + 6\csc\left(-\frac{1}{4}\theta + 15°\right)$

4. Graph manually: $y = 5 - 3\tan\frac{1}{2}(x - \pi)$

II. Writing Equations:

Examples:

1. Amplitude = 2, Period = 90°, phase shift = 45° and translation = 4

2. As you ride a Ferris wheel, the height that you are above the ground varies periodically. Consider the height of the center of the wheel to be the equilibrium point. A particular wheel has a diameter of 38-ft., travels at a rate of 4 revolutions per minute, and the seats of the Ferris wheel clear the ground by 3-ft.

   a) Write an equation to describe the change in height, *h*, of the seat that was filled last before the ride began in terms of time, *t*, in seconds.

   b) Find the height of the seat after 22-s.

Homework: worksheets

Section 6.5C – Graphs of Compositions

Objectives:
1. To graph composition functions using the t-bar method.

A. $y = \sin\theta$

θ	y	θ	y	θ	y	θ	y
0				180			
30		120		210		300	
45		135		225		315	
60		150		240		330	
90				270			

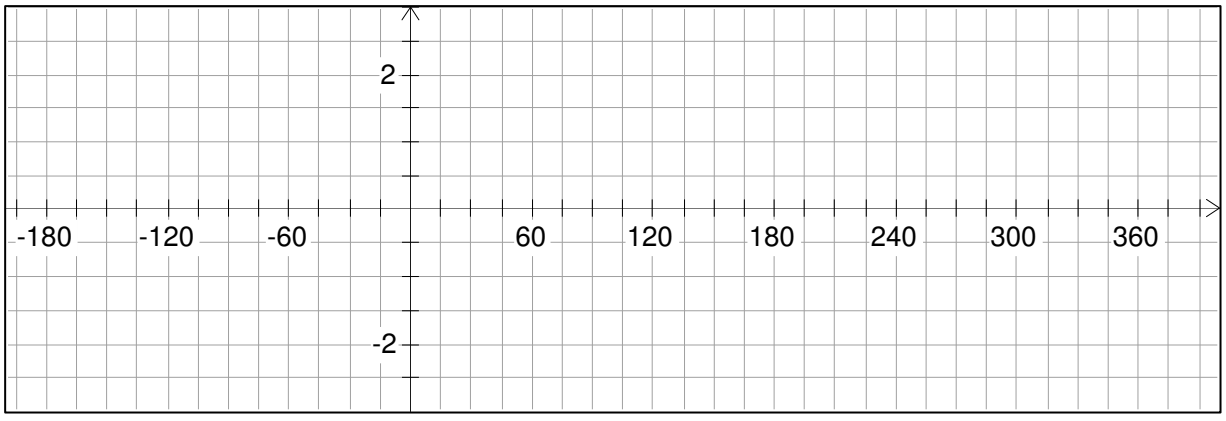
B. $y = \cos\theta$

θ	y	θ	y	θ	y	θ	y
0				180			
30		120		210		300	
45		135		225		315	
60		150		240		330	
90				270			

C. $y = \sin\theta + \cos\theta$

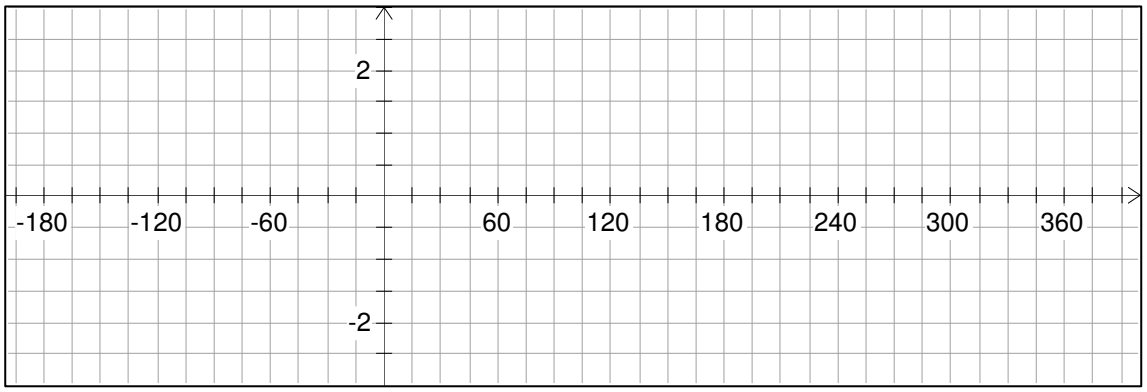
θ	y	θ	y	θ	y	θ	y
0				180			
30		120		210		300	
45		135		225		315	
60		150		240		330	
90				270			

Graph:



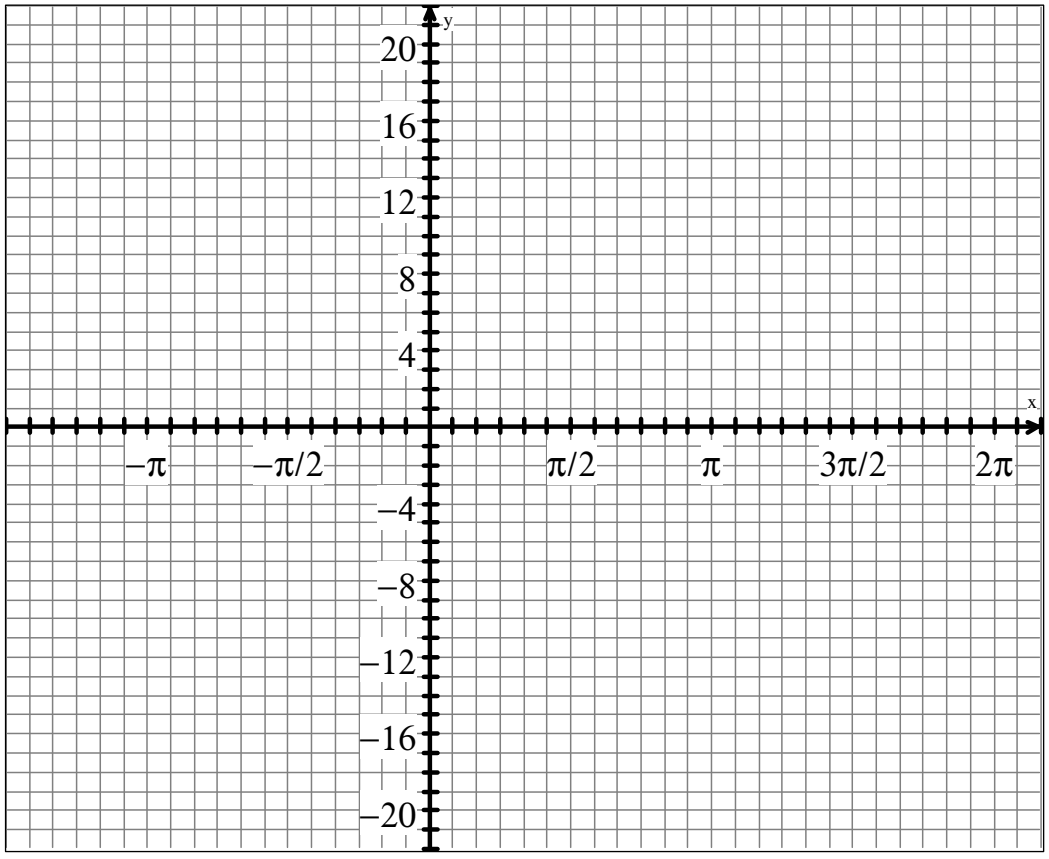
D. $y = \sin\theta\cos\theta$

Graph:



E. $y = 3x - \sin x$

x	y	x	y	x	y	x	y
0				π			
$\pi/6$		$5\pi/6$		$7\pi/6$		$11\pi/6$	
$\pi/4$		$3\pi/4$		$5\pi/4$		$7\pi/4$	
$\pi/3$		$2\pi/3$		$4\pi/3$		$5\pi/3$	
$\pi/2$				$3\pi/2$			



Homework: p.383 – 36-38 all

Section 6.8 – Principle Values of Inverse Trig Functions

Objectives:
1. To find the principle values of inverse trig functions.

A. Principle Values:

1. $\theta = Sin^{-1}y \ \ (\theta = Arc\sin y)$ _____ $\le \theta \le$ _____
   $x = Sin^{-1}y \ \ (x = Arc\sin y)$ _____ $\le x \le$ _____
2. $\theta = Cos^{-1}y \ \ (\theta = Arc\cos y)$ _____ $\le \theta \le$ _____
   $x = Cos^{-1}y \ \ (x = Arc\cos y)$ _____ $\le x \le$ _____
3. $\theta = Tan^{-1}y \ \ (\theta = Arc\tan y)$ _____ $< \theta <$ _____
   $x = Tan^{-1}y \ \ (x = Arc\tan y)$ _____ $< x <$ _____

Note: Capital letters are used to distinguish the function with restricted domains from the usual non-restricted trig functions.

B. Examples

1. $Arc\sin\left(-\frac{1}{2}\right)$

2. $Tan^{-1}\left(\sin\frac{\pi}{2}\right)$

Note: Be careful – $\left\{\sin\left(Sin^{-1}x\right) = x\right\}$ but $\left\{Sin^{-1}\left(\sin x\right)\right\}$ does not necessarily equal x. (example: $2\pi/3$)

3. $\tan\left(Arc\cos\frac{1}{2} - Arc\sin\frac{1}{2}\right)$

Homework: p.410 – 8-10 all, 22-33 all, 37, 46-50 all, 53, 54

Section 6.8B – Simple Harmonic Functions

Objectives:
1. To find the general equations to solve simple harmonic function.
2. To solve simple harmonic functions.

General Equations

A. $\theta = cos^{-1} y \ \ (\theta = arc\cos y)$

$x = cos^{-1} y \ \ (x = arc\cos y)$

1. Graph: $y = \cos x$
2. General equations:

B. $\theta = sin^{-1} y \ \ (\theta = arc\sin y)$

$x = sin^{-1} y \ \ (x = arc\sin y)$

1. Graph: $y = \sin x$
2. General equations:

C. $\theta = tan^{-1} y \ \ (\theta = arc\tan y)$

$x = tan^{-1} y \ \ (x = arc\tan y)$

1. Graph: $y = \tan x$
2. General equation:

D. Examples

1. Find the values of x in the interval $0 \le x \le 2\pi$ that satisfy each equation.

a) $x = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

b) $x = \arctan 2.1445$

HW: Worksheet

2. Find all the first three positive values of t that satisfy the equation: $27 = 21 - 19\cos\frac{2\pi}{15}t$

3. Find all the values of θ that satisfy the equation: $\frac{1}{2} = 1 - \sin 3(\theta - 90°)$

4. The equation $y = 54.5 + 23.5\sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right)$ models the average monthly temperatures of Springfield, Missouri. In this equation, t denotes the number of months with January represented by 1. During which two months is the average temperature 54.5°?

Homework: worksheet